

LECTURE NOTE
On
STRUCTURAL MECHANICS
(3RD SEM. Civil Engineering)

Prepared By:

SRI PADMA LOCHAN BEHERA

SR.LECTURER (Civil Engg.)



Government Polytechnic, Sonapur

Structural Mechanics

Mechanical ppts of material →

The characteristics of material that describe the behavior under the action of external loads are referred as its mechanical ppts. The common mechanical ppts are as follows.

Strength →

→ It is defined as ability of material to resist loads without failure.

→ It is usually expressed or measured in terms of maximum load per unit area (i.e. max^m stress or ultimate strength) that a material can withstand failure and it varies according to the type of loading. Further the strength is divided into three types they are.

① Tensile strength →

→ The tensile strength or tenacity is defined as the ability of material to resist a stretching (tensile) load without fracture.

② Compressive strength →

The ability of a material to resist squeezing (compressive) load without fractures called compressive strength.

③ Shear strength →

The ability of a material to resist transverse loads, i.e. loads tending to separate (or cut) the material is called shear strength.

Rigidity →

Rigidity is defined as the ppt exhibited by the solid to change in its shape. That is when an external force is applied to the solid material, there won't be any change in the shape. This shows that the particles are closely packed and the attraction betⁿ these particles is very strong.

Elasticity →

The ability of material to deform under load and return to its original shape when the load is removed is called elasticity.

Plasticity →

The ability of a material to deform under load and retain its new shape when the load is removed is called plasticity.

Compressibility →

→ compressibility may be defined as the ratio of change in volume of the fluid to the original volume. It can be mathematically expressed as

$$(\text{Change in volume} / \text{original volume}) \times 100$$

→ It is often represented in percentage that is why ^{we} are multiplying it with 100.

Hardness →

It is defined as the ability of a material to resist scratching or indentation by another hard body. Hardness is directly related to strength.

Toughness →

It is defined as the ability of the material to absorb energy up to fracture during the plastic deformation. Toughness of a metal offers the resistance to breaking when force is applied.

Stiffness →

→ It is the ability of material to resist deformation or deflection under load.

→ within the elastic limit, stiffness is measured by the modulus of elasticity.

Brittleness →

It is the ppt of sudden fracture without any visible permanent deformation.

Ductility →

It is the ability of a material to be deformed plastically without rupture under tension. It can be drawn out into fine wire without fracture.

Malleability →

→ It is the ability of a material to be deformed plastically without rupture under compressive load.

→ Due to this ppt metals are hammered and rolled into thin sheets.

Creep →

The slow and progressive deformation of material with time at constant stress is called creep.

Fatigue →

Failure of material under repeated or reversal stress is called fatigue. Machine parts are frequently subjected to varying stress and it is imp. to know the strength of fatigue limit.

Tenacity →

Tenacity is defined as the specific stress corresponding to the max^m force on a force extension curve.

Durability →

Durability may be defined as the ability of a structure to remain fit for its intended or foreseen use throughout its design working life, with an appropriate level of maintenance.

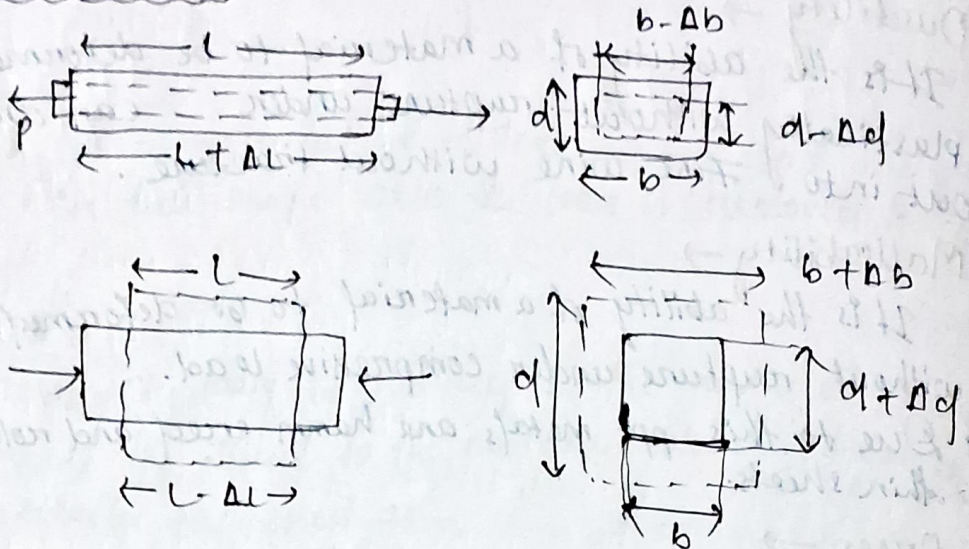
Resilience →

It is ppt of material to absorb energy and to resist shocks and impact loads. It is measured by the amount of energy absorbed per unit volume within the elastic limit.

04-04/08/23

Strain $\rightarrow$ result of stress

Longitudinal strain $\rightarrow$



- $\rightarrow$ longitudinal strain is defined as the ratio of change in length to original length.
- $\rightarrow$ It is denoted as ϵ (Epsilon) / $\epsilon_{\text{long}} = \frac{\Delta L}{L}$
- $\rightarrow$ Strain is a Unitless quantity.

Lateral Strain $\rightarrow$

- $\Rightarrow$ It is the ratio of change in lateral dimension that is breadth / depth is known as lateral strain.
- $\Rightarrow$ It is denoted as $\epsilon_{\text{lateral}} = \frac{\Delta b}{b}$ or $\frac{\Delta d}{d}$

Hooke's law $\rightarrow$

- $\rightarrow$ It states that within the limit of elasticity stress is directly proportional to strain.

$$\text{Stress} \propto \text{strain} \quad \sigma \propto \epsilon$$

$$\text{Stress} = E \text{ strain} \quad \sigma = E \epsilon$$

where,

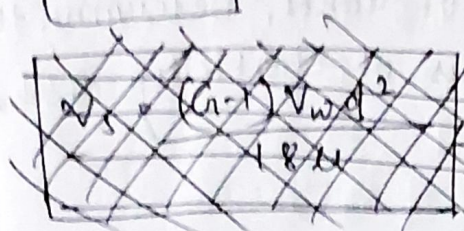
$E =$ Modulus of Elasticity

Or Young's Modulus

$$E = \frac{\sigma}{\epsilon}$$

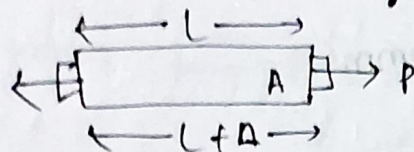
Unit of young's Modulus is N/mm^2 / MPa

06.7/09/23



$$W_w = 0.81 \text{ kNm}^3$$

Deformation of a body subjected to Load →



P = Applied load

L = Original length of the object

A = Area of the object

$L + \Delta$ = Final length

Deformation = Final length - Original length
(Δ)

$$\epsilon = \frac{\Delta L}{L} \text{ (strain)}$$

$$\sigma = \frac{P}{A} \text{ (stress)}$$

$$E = \frac{\sigma}{\epsilon} = \frac{\frac{P}{A}}{\frac{\Delta L}{L}} = \frac{P}{A} \times \frac{L}{\Delta L} = \frac{PL}{A\Delta L}$$

$$E = \frac{PL}{A\Delta L}$$

$$\Delta L = \frac{PL}{AE}$$

E = Modulus of Elasticity /
young's Modulus.

- ① A steel rod of length 1m. and cross-section 20mm x 20mm is subjected to a tensile force of 40kN. Determine the elongation of the rod if modulus of elasticity of the material is 200GPa?

Ans length of a steel rod = 1m.

$$1\text{m} = 1000\text{mm}.$$

$$\text{Area} = 20\text{mm} \times 20\text{mm} = 400\text{mm}^2$$

$$P = 40\text{kN} = 40 \times 10^3\text{N}$$

$$E = 200\text{GPa} = 200 \times 10^3\text{MPa}$$

$$\downarrow$$

$$10^9$$

$$\Delta = \frac{PL}{AE} = \frac{40 \times 10^3 \times 1000}{400 \times 200 \times 10^3} = 0.5\text{mm}.$$

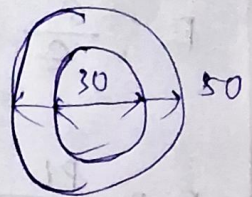
- ② A hollow cylinder of length 2m. has outside diameter 50mm and inside diameter 30mm is subjected to a load of 25kN. Find the stress in the cylinder and the deformation of the cylinder if modulus of elasticity is 100GPa.

Ans length $l = 2\text{m} = 2 \times 10^3\text{mm}$

$D = 50\text{mm}$ (outside diameter)

$d = 30\text{mm}$ (inside diameter)

$$P = 25\text{kN} = 25 \times 10^3\text{N}$$



So, stress (σ) = ? ~~or~~ ~~deformation~~ Δ = ? (deformation of cylinder)

$$E = 100\text{GPa} = 100 \times 10^9\text{N/m}^2$$

~~$$E = 100 \times 10^3\text{N}$$~~
~~$$10^3 \times 10^3\text{mm}^2$$~~

$$E = 100 \times 10^9\text{N}$$

$$\frac{10^3 \times 10^3\text{mm}^2}{\text{mm}^2}$$

~~$$E = 100 \times 10^3\text{N}$$~~

$$E = 100 \times 10^3\text{N}$$

$$\frac{\text{mm}^2}{\text{mm}^2}$$

Area of hollow cylinder $A = \frac{\pi}{4} (D^2 - d^2)$



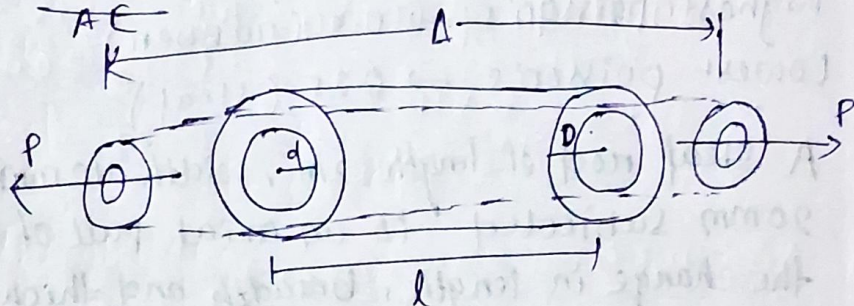
$$A = \frac{\pi}{4} (50^2 - 30^2)$$

$$A = \frac{\pi}{4} (2500 - 900)$$

$$A = 1256.64$$

$$A = 1257 \text{ mm}^2$$

we know, $\Delta = \frac{Pl}{AE}$



we know, $\Delta = \frac{Pl}{AE}$

stress (σ) = $\frac{P}{A} \text{ N/mm}^2$

$$\Delta = \frac{25 \times 10^3 \times 2 \times 10^3}{1257 \times 100 \times 10^3}$$

$$\sigma = \frac{25 \times 10^3}{1257}$$

$$\sigma = 19.9 \text{ N/mm}^2$$

$$\Delta = 0.4 \text{ mm} \quad \underline{\text{Ans}}$$

Ans

Trick Significant rule

5.65789

After 2 digit 3rd num $\rightarrow$ greater than 5 $\rightarrow +1$

Smaller than 5 $\rightarrow -1$

then 5.66789

If even num $\rightarrow$ same num

but odd num $\rightarrow +1$ add

Elastic Constants →

$$E = \frac{\sigma}{\epsilon}$$

→ Poisson's ratio is an elastic constant which is the ratio of lateral strain to longitudinal strain.

~~Elastic constant~~ → It is denoted as ' μ '.

$$\mu = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

- highest Poisson's → 0.5 (rubber)
- lowest Poisson's → 0.25 (steel)

(Q) A steel rod of length 2m, width 40mm and thickness 20mm subjected to an axial pull of 160kN. Determine the change in length, breadth and thickness if modulus of elasticity is 200GPa and Poisson's ratio 0.3? (L.Q) 10 marks

Ans Given, length = 2m = 2×10^3 mm

width = 40mm (b)

thickness = 20mm (d)

P = 160kN = 160×10^3 N

E = 200GPa = 200×10^3 MPa

$\mu = 0.3$

then $\Delta l, \Delta b, \Delta d = ?$

$$\Delta = \frac{PL}{AE} = \frac{160 \times 10^3 \times 2 \times 10^3}{40 \times 20 \times 200 \times 10^3} = 2 \text{ mm}$$

$$\mu = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$\text{longitudinal strain} = \frac{\Delta l}{l}$$

$$= \frac{2}{2000} = 0.001$$

$$\Rightarrow 0.3 = \frac{\text{Lateral strain}}{0.001}$$

$$\Rightarrow \text{Lateral strain} = 0.3 \times 0.001 = 0.0003$$

$$\text{Lateral strain} = \frac{\Delta b}{b}$$

$$\Rightarrow 0.0003 = \frac{\Delta b}{40}$$

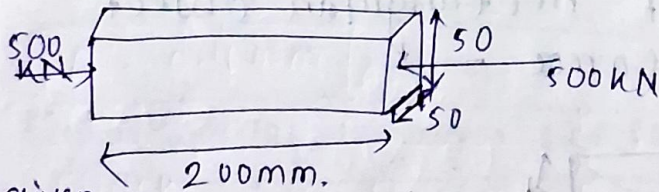
$$\Rightarrow \Delta b = 0.0003 \times 40 = 0.012 \text{ mm.}$$

$$\text{Lateral strain} = \frac{\Delta d}{d}$$

$$\Rightarrow 0.0003 = \frac{\Delta d}{20}$$

$$\Rightarrow \Delta d = 0.0003 \times 20 = 0.006 \text{ mm.}$$

(Q) A metal bar 50mm x 50mm in section is subjected to an axial compressive load of 500kN. If the contraction of a 200mm gauge length was found to be 0.5 mm and the increase in thickness 0.04 mm, find the values of young's modulus and poisson's ratio for the bar material?



Any given $\rightarrow$

$$A = 50 \times 50 = 2500 \text{ mm}^2$$

$$P = 500 \text{ kN} = 500 \times 10^3 \text{ N}$$

$$L = 200 \text{ mm.}$$

$$\Delta L = \frac{PL}{AE}$$

$$\Rightarrow 0.5 = \frac{500 \times 10^3 \times 200}{2500 \times E}$$

$$\Delta L = 0.5 \text{ mm}$$

$$\Delta d = 0.04 \text{ mm}$$

$$\Rightarrow E = \frac{500 \times 10^3 \times 200}{2500 \times 0.5}$$

$$\mu = ?$$

$$= 80000 \text{ N/mm}^2 \text{ or MPa}$$

$$\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{\Delta d}{d} \div \frac{\Delta l}{l}$$

$$\mu = \frac{\frac{\Delta d}{d}}{\frac{\Delta l}{l}}$$

$$\text{Lateral strain} = \frac{\Delta d}{d} = \frac{0.04}{50} = 8 \times 10^{-4}$$

$$\text{Longitudinal strain} = \frac{\Delta l}{l} = \frac{0.5}{200} = 2.5 \times 10^{-3}$$

$$\therefore \mu = \frac{8 \times 10^{-4}}{2.5 \times 10^{-3}} = 0.32$$

Q1. 03/10/23

Q) Volumetric strain →

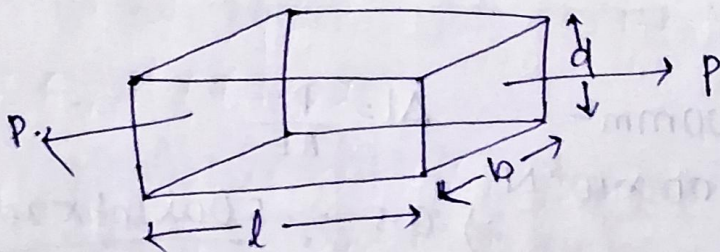
Change in volume to original volume is known as volumetric strain.

$$\epsilon_v = \frac{\Delta v}{v}$$

Δv = change in volume

v = Original volume

Volumetric strain of rectangular object subjected to axial force →



P = Axial force

l = length of object

b = width

d = depth.

δl = change in length

δb = change in breadth

δd = change in depth

$$v = l \times b \times d$$

$$\begin{aligned}\text{Final volume} &= (l + \delta l)(b - \delta b)(d - \delta d) \\ &= lbd + \delta l \cdot bd - l\delta b d - l b \delta d\end{aligned}$$

$$\text{change in volume} = lbd + \delta l$$

$$\begin{aligned}\text{change in volume} &= lbd + \delta lbd - l\delta b d - l b \delta d - lbd \\ &= \delta lbd - l\delta b d - l b \delta d\end{aligned}$$

$$E_v = \frac{\text{change in volume}}{\text{original volume}}$$

$$\begin{aligned}E_v &= \frac{\delta lbd - l\delta b d - l b \delta d}{lbd} \\ &= \frac{\delta l}{l} - \frac{\delta b}{b} - \frac{\delta d}{d} \quad \text{or}\end{aligned}$$

$$E_v = \frac{\Delta l}{l} - \frac{\Delta b}{b} - \frac{\Delta d}{d}$$

$$E_v = \text{Longitudinal strain} - \text{Lateral strain} \times 2$$

$$\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\Rightarrow \boxed{\text{Lateral strain} = \mu \times \text{long. strain}}$$

$$\begin{aligned}\therefore E_v &= \text{long. strain} - \text{lateral strain} \times 2 \\ &= \text{long. strain} - 2\mu \times \text{long. strain}\end{aligned}$$

$$E_v = \text{long. strain} (1 - 2\mu)$$

$$\therefore \boxed{E_v = e (1 - 2\mu)} \quad \text{IMP}$$

(Q) A steel bar of length 1.2 m, width 50 mm and thickness 40 mm is subjected to an axial pull of 150 kN. Determine the change in volume if $E = 200 \text{ GPa}$ and $\mu = 0.25$.

Ans Given data,
length (L) = 1.2 m = $1.2 \times 10^3 \text{ mm}$
= 1200 mm

$$P = 150 \text{ kN} = 150 \times 10^3 \text{ N}$$

$$\text{width } (b) = 50 \text{ mm}$$

$$\text{thickness } (d) = 40 \text{ mm}$$

$$E = 200 \text{ GPa}$$

$$\mu = 0.25$$

change in volume / volumetric strain = ?

$$\Delta L = \frac{PL}{AE} = \frac{150 \times 10^3 \times 1.2 \times 10^3}{50 \times 40 \times 200 \times 10^3}$$

$$\text{Area } (A) = b \times d$$

$$\text{Area } (A) = b \times d$$

$$\Delta L = 0.45 \text{ mm}$$

$$\text{Longitudinal strain } (\epsilon) = \frac{\Delta L}{L}$$

$$= \frac{0.45}{1.2 \times 10^3} = 3.75 \times 10^{-4}$$

$$\therefore \epsilon_v = \epsilon(1 - 2\mu)$$

$$= 3.75 \times 10^{-4} (1 - 2 \times 0.25)$$

$$= 1.85 \times 10^{-4}$$

$$\epsilon_v = \frac{\text{change in volume}}{\text{Original volume}} = \frac{\delta v}{v}$$

$$\delta v = \epsilon_v \times v$$

$$= 1.85 \times 10^{-4} \times 1.2 \times 10^3 \times 50 \times 40$$

$$= 444 \text{ mm}^3$$

Bulk Modulus $\rightarrow$

When a material is subjected to 3 mutually perpendicular stresses in 3 dirⁿ.

The ratio of direct stress to volumetric strain remains constant which is known as Bulk modulus.

$\rightarrow$ It is denoted by ' K '.

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}}$$

$$K = \frac{\sigma}{\epsilon_v}$$

Relation betⁿ K and E .

Let a cube have l, b, d

E = modulus of Elasticity

$$\frac{\delta l}{l} = \frac{\sigma}{E} (1 - 2\mu)$$

$$\text{If } l = b = d = l$$

$$E_v = E(1 - 2\mu)$$

$$\Rightarrow \frac{\delta v}{v} = \frac{\sigma}{E} (1 - 2\mu)$$

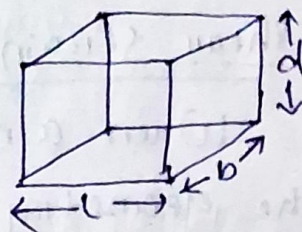
$$v = l \times b \times d = l^3$$

$$\Rightarrow \frac{\delta v}{v} = 3l^2$$

$$\Rightarrow \delta v = 3l^2 \times \delta l$$

$$\Rightarrow \frac{3l^2 \cdot \delta l}{l^3} = \frac{\sigma}{E} (1 - 2\mu)$$

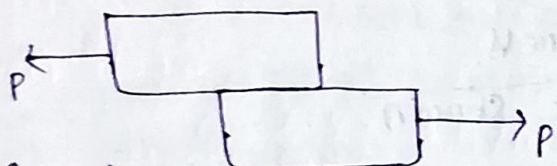
$$\text{Final formula} \rightarrow \boxed{E = 3K(1 - 2\mu)}$$



Q) Shear stress →

21.05/20/23

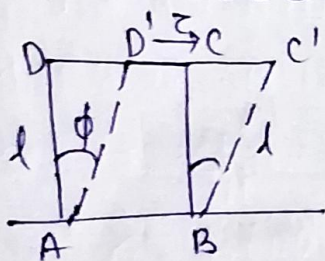
When there are two forces acting at two different directions, they will try to shear-off the object and during that process a stress is developed which is known as shear stress.



- It is denoted as τ (tau)
- Unit of shear stress is N/mm^2

Q) Shear strain →

When a material is subjected to shear stress, the geometry of the material changes which produces an angular displacement, which is called shear strain.



$$\phi = \frac{DD'}{AD}$$

$$= \frac{DD'}{l}$$

Q) Modulus of Rigidity (Shear modulus) →

For every material the ratio of shear stress to shear strain always remains constant, which is called modulus of rigidity (shear modulus).

- It is denoted by 'G'.

$$G = \frac{\tau}{\phi}$$

$$\text{Modulus of rigidity} = \frac{\text{Shear stress}}{\text{Shear strain}}$$

Relation betⁿ E and G $\rightarrow$ (Derived Question)

$$E = 2G(1 + \mu)$$

$$\Rightarrow \frac{E}{2G} = 1 + \mu$$

$$\Rightarrow \mu = \frac{E}{2G} - 1$$

$$\bullet \boxed{E = 3K(1 - 2\mu)}$$

Put the value of ' μ ' in 'E'

$$\Rightarrow E = 3K \left\{ 1 - 2 \left(\frac{E}{2G} - 1 \right) \right\}$$

$$\Rightarrow E = 3K \left(1 - \frac{E}{G} + 2 \right)$$

$$\Rightarrow E = 3K \left(3 - \frac{E}{G} \right)$$

$$\Rightarrow E = 9K - \frac{3KE}{G}$$

$$\Rightarrow E + \frac{3KE}{G} = 9K$$

$$\Rightarrow E \left(1 + \frac{3K}{G} \right) = 9K$$

$$\Rightarrow E \times \frac{G + 3K}{G} = 9K$$

$$\Rightarrow \boxed{E = \frac{9KG}{3K + G}}$$

(Q) The modulus of rigidity of a material is $0.8 \times 10^5 \text{ N/mm}^2$. When a $6\text{mm} \times 6\text{mm}$ rod of this material was subjected to an axial pull of 3600N , it was found that the lateral dimension of the rod changed to $5.9991 \times 5.9991\text{mm}$. Find the poisson's ratio and the modulus of elasticity!

Ans Given data,

$$G = 0.8 \times 10^5 \text{ N/mm}^2$$

$$P = 3600 \text{ N}$$

$$b \times d = 6 \times 6 \text{ mm}$$

$$\text{Final } b \times d = 5.9991 \times 5.9991 \text{ mm.}$$

$$\begin{aligned} \delta b, \delta d &= 6 - 5.9991 \\ &= 0.0009 \text{ mm.} \end{aligned}$$

$$\begin{aligned} \text{Lateral strain} &= \frac{\delta b}{b} = \frac{0.0009}{6} \\ &= 0.00015 \end{aligned}$$

$$\text{Poisson's ratio } (\mu) = ?$$

$$\text{Modulus of elasticity } (E) = ?$$

$$\sigma = \frac{P}{A}$$

$$\sigma = \frac{3600}{6 \times 6} = 100 \text{ N/mm}^2$$

$$E = \frac{\text{Stress}}{\text{Strain}}$$

$$= \frac{100}{0.00015}$$

$$= 666666.67 \text{ MPa}$$

$$E = 200(1 + \mu)$$

$$666666.67 = 2 \times 0.8 \times 10^5 (1 + \mu)$$

$$1 + \mu = \frac{2 \times 0.8 \times 10^5}{666666.67}$$

$$1 + \mu = 4.168$$

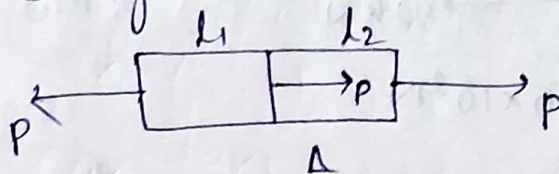
$$\mu = 4.168 - 1$$

$$\mu = 3.168$$

01.06/10/23

Principle of Superposition →

When an object is subjected to forces at different locations then the total deformation ~~then total deformation~~ of the object will be the algebraic sum of the individual deformation.



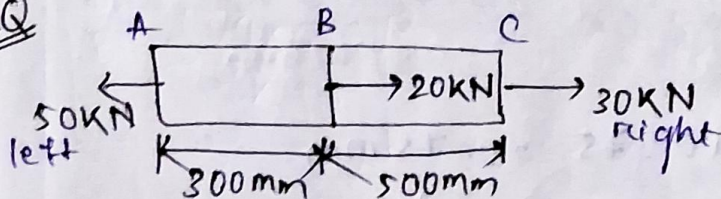
$$\Delta = \frac{Pl}{AE}$$

$$\Delta_1 = \frac{Pl_1}{A_1 E}$$

$$\Delta_2 = \frac{Pl_2}{A_2 E}$$

$$\Delta = \Delta_1 + \Delta_2$$

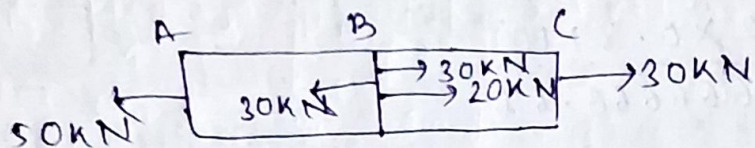
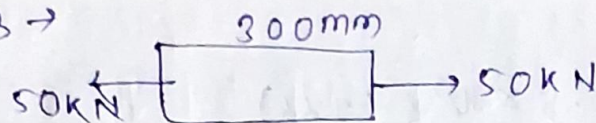
1Q



A steel bar of ~~ex~~ cross-sectional ~~bar~~ area of 200mm^2 is loaded as shown in fig. Determine the total elongation of bar taking $E = 200\text{GPa}$

Ans

AB $\rightarrow$



$$P_1 = 50 \text{ kN} = 50 \times 10^3 \text{ N}$$

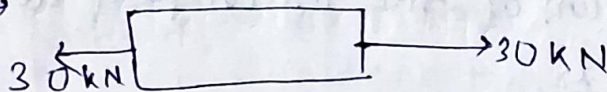
$$A = 200 \text{ mm}^2$$

$$L_1 = 300 \text{ mm}$$

$$E = \frac{200 \times 10^3}{200 \text{ GPa}} = 200 \times 10^3 \text{ MPa}$$

$$\Delta_1 = \frac{P_1 L_1}{AE} = \frac{50 \times 10^3 \times 300}{200 \times 200 \times 10^3} = 0.375 \text{ mm}$$

BC $\rightarrow$



$$P_2 = 30 \text{ kN} = 30 \times 10^3 \text{ N}$$

$$A = 200 \text{ mm}^2$$

$$L_2 = 500 \text{ mm}$$

$$E = 200 \times 10^3 \text{ MPa}$$

$$\Delta_2 = \frac{P_2 L_2}{AE} = \frac{30 \times 10^3 \times 500}{200 \times 200 \times 10^3} = 0.375 \text{ mm}$$

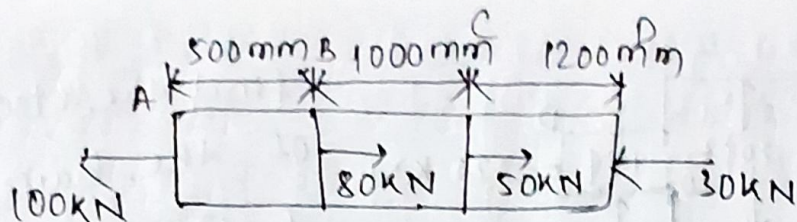
$$\therefore \Delta_1 = 0.375 \text{ mm}$$

$$\Delta_2 = 0.375 \text{ mm}$$

$$\Delta = \Delta_1 + \Delta_2 = 0.375 + 0.375 = 0.75 \text{ mm}$$

Ans

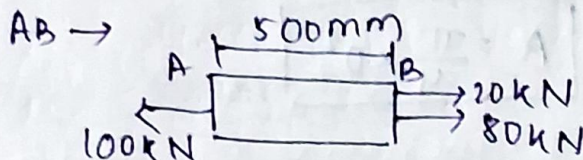
2



$$A = 500 \text{ mm}^2$$

$$E = 80 \text{ GPa}$$

Ans



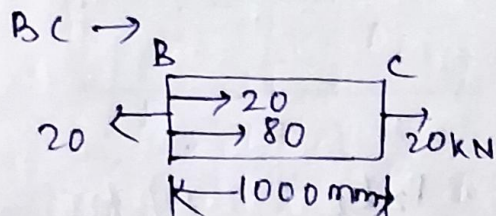
$$P_1 = 100 \text{ kN} = 100 \times 10^3 \text{ N}$$

$$A = 500 \text{ mm}^2$$

$$L_1 = 500 \text{ mm}$$

$$E = 80 \text{ GPa} = 80 \times 10^3 \text{ MPa}$$

$$\Delta_1 = \frac{P_1 L_1}{AE} = \frac{100 \times 10^3 \times 500}{500 \times 80 \times 10^3} = 1.25 \text{ mm}$$



$$P_2 = 20 \text{ kN} = 20 \times 10^3 \text{ N}$$

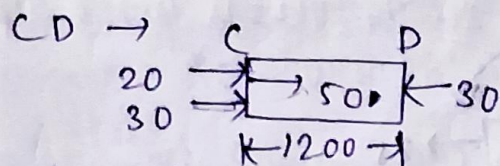
$$L_2 = 1000 \text{ mm}$$

$$A = 500 \text{ mm}^2$$

$$E = 80 \times 10^3 \text{ MPa}$$

$$\Delta_2 = \frac{P_2 L_2}{AE}$$

$$= \frac{20 \times 10^3 \times 1000}{500 \times 80 \times 10^3} = 0.5 \text{ mm}$$



$$P_3 = 30 \times 10^3 \text{ N} \quad L_3 = 1200 \text{ mm}$$

$$A = 500 \text{ mm}^2 \quad E = 80 \times 10^3 \text{ MPa}$$

$$\Delta_3 = \frac{P_3 L_3}{AE} = \frac{30 \times 10^3 \times 1200}{500 \times 80 \times 10^3}$$

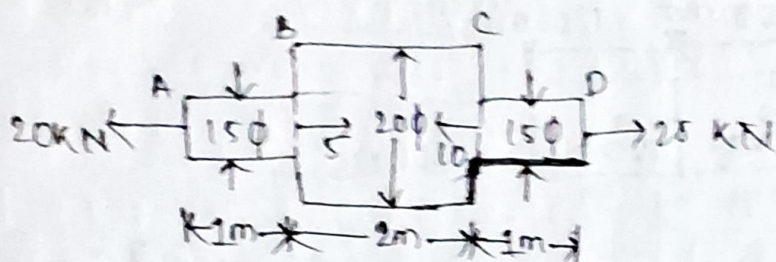
$$= 0.9 \text{ mm}$$

$$\therefore \Delta = \Delta_1 + \Delta_2 - \Delta_3$$

$$= 1.25 + 0.5 - 0.9$$

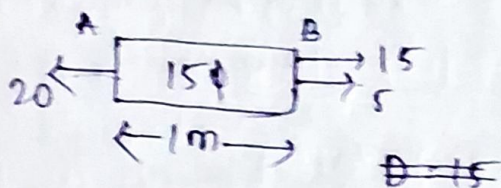
$$= 0.85 \text{ mm} \quad \text{Ans}$$

3



Find the elongation of the bar if $E = 200 \text{ GPa}$

Ans AB $\rightarrow$



$$A = \frac{\pi}{4} \times D^2$$

$$A_1 = \frac{\pi}{4} \times (15)^2 = 176.71 \text{ mm}^2$$

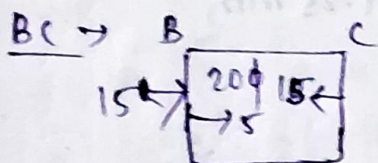
$$\Delta_1 = \frac{P_1 L_1}{A_1 E}$$

$$P_1 = 20 \text{ kN} = 20 \times 10^3 \text{ N}$$

$$L_1 = 1 \text{ m} = 1000 \text{ mm}$$

$$E = 200 \text{ GPa} = 200 \times 10^3 \text{ MPa}$$

$$= \frac{20 \times 10^3 \times 1000}{176.71 \times 200 \times 10^3} = 0.565 \text{ mm}$$



$$A_2 = \frac{\pi}{4} \times (20)^2$$

$$= 314.15 \text{ mm}^2$$

$$P_2 = 15 \text{ kN} = 15 \times 10^3 \text{ N}$$

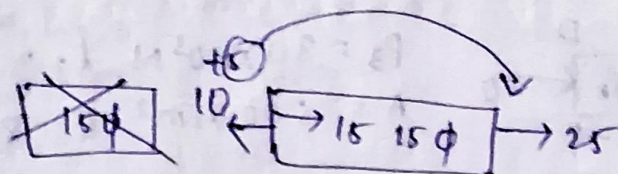
$$L_2 = 2 \text{ m} = 2000 \text{ mm}$$

$$E = 200 \times 10^3 \text{ MPa}$$

$$\Delta_2 = \frac{P_2 L_2}{A_2 E}$$

$$= \frac{15 \times 10^3 \times 2000}{314.15 \times 200 \times 10^3} = 0.477 \text{ mm}$$

CD $\rightarrow$



$$A_3 = \frac{\pi}{4} \times (15)^2 = 176.71 \text{ mm}^2$$

$$L_3 = 1 \text{ m} = 1000 \text{ mm}$$

$$P_3 = 25 \text{ kN} = 25 \times 10^3 \text{ N}$$

$$E = 200 \times 10^3 \text{ MPa}$$

$$\Delta_3 = \frac{P_3 l_3}{A_3 E} = \frac{25 \times 10^3 \times 1000}{176.71 \times 200 \times 10^3} = 0.707 \text{ mm}$$

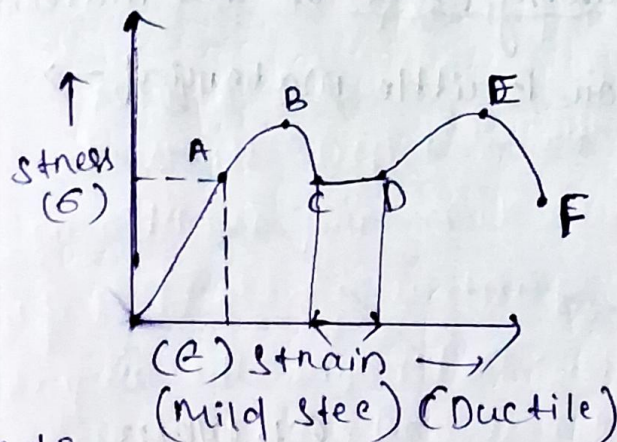
$$\therefore \Delta = \Delta_1 + \Delta_2 + \Delta_3 = 0.565 + 0.477 + 0.707 = 1.749 \text{ mm}$$

Q. Stress-Strain Curve →

15

Qt. 09/10/23

For any material the relation betn stress and strain is represented to a curve graph which is called stress-strain curve.



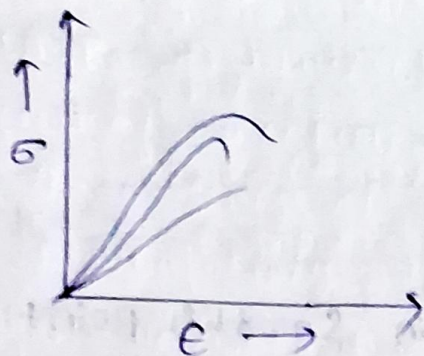
Up to
→ Point A → the stress & strain so this point is called proportionality limit or elastic limit.

→ Point B → From point A to B there is increase in stress the strain increases gradually and when it reaches the point 'B' the yielding of material starts. So point 'B' is called upper yield point.

→ Point C → From point B to C the stress reduces but the yielding continues, so the strain increases and point 'C' is called lower yield point.

- Point D → From point C to D there is increase in strain without any change in stress which is called strain hardening.
- Point E → From point D to E the stress again increases with the increase in strain and it reaches the max^m value at point 'E', which is called ultimate point.
- Point F → After point E the stress gradually reduces but the strain increases and up to point 'F', at which the material breaks, which is called the breaking point of the material.

⇒ Stress-strain curve for Brittle material →



01.10/10/23

⇒ Principal Stress → Plane →

In a strain^{ed} material there are 3 planes mutually perpendicular to each other, which carries only direct stress and no shear stress are called Principal plane.

- The stress which is developed along the principal plane is called principal stress.

→ There are two methods to calculate the principal stress in a material.

① Analytical method

② Graphical method / Mohr's Circle method

↓ (scientist of how to calculate the principal stress)

① Analytical Method → (10 Marks)

Condition-1 →

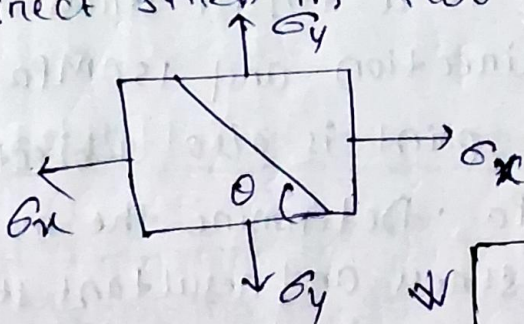
The material subjected to direct stress only in one dirn.



* Any inclined plane to angle θ

Condition-2 →

The material subjected to direct stress in two dirn.



$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta$$

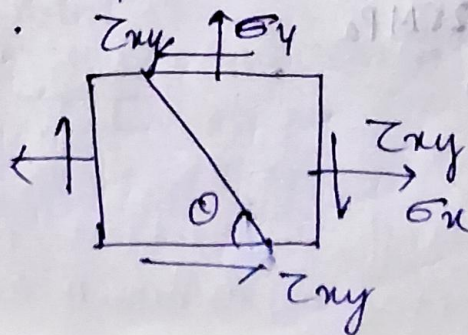
$$\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta$$

$$\sigma_R = \sqrt{\sigma_n^2 + \tau^2}$$

Resultant stress

Condition-3 →

The material subjected to direct stress and Shear stress.



$$\Downarrow \sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\Downarrow \tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta \quad \Downarrow$$

$$\Downarrow \tau_R = \sqrt{\sigma_n^2 + \tau^2} \quad (\text{Resultant shear stress})$$

$$\sigma_{p_1} = \left(\frac{\sigma_x + \sigma_y}{2} \right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

(Max^m principal stress)

$$\sigma_{p_2} = \left(\frac{\sigma_x + \sigma_y}{2} \right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

(Min^m principal stress)

Dt. 13/10/23

(Q) A point is subjected to a tensile stress of 250 MPa in horizontal direction and 150 MPa in vertical direction, the point is also subjected to a shear stress of 25 MPa. Determine the magnitude of normal stress, shear stress and resultant stress. Along an inclined plane at an angle of 20° to the horizontal axis.

Ans Given data,

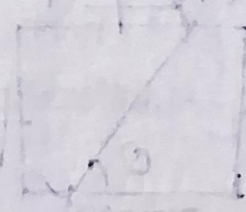
$$\sigma_x = 250 \text{ MPa}$$

$$\sigma_y = 150 \text{ MPa}$$

$$\tau_{xy} = 25 \text{ MPa}$$

$$\theta = 20^\circ$$

Ans:



$$\begin{aligned}\sigma_n &= \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta \\ &= \frac{250 + 150}{2} - \frac{250 - 150}{2} \cos 2 \times 20 - 25 \sin 2 \times 20 \\ &= 145.62 \text{ MPa}\end{aligned}$$

$$\begin{aligned}\tau &= \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta \\ &= \frac{250 - 150}{2} \sin (2 \times 20) - 25 \times \cos (2 \times 20) \\ &= 12.98 \text{ MPa}\end{aligned}$$

$$\begin{aligned}\tau_R &= \sqrt{(\sigma_n)^2 + (\tau)^2} \\ &= \sqrt{(145.62)^2 + (12.98)^2} \\ &= 146.19\end{aligned}$$

01.16/10/23

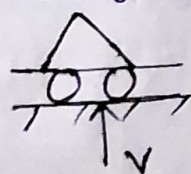
Q) Types of Load and support →

Types of Support →

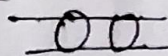
Depending on the reaction criteria support is of 3 types.

① Roller Support →

The support which restricts the vertical motion only and allows the horizontal motion and bending is called 'roller support'.

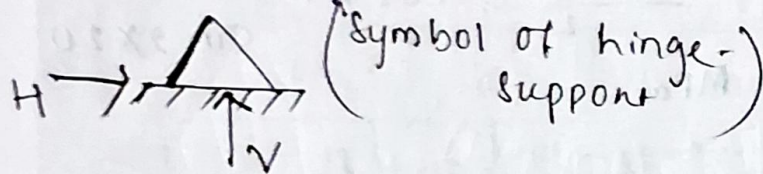


(Symbol of roller support)



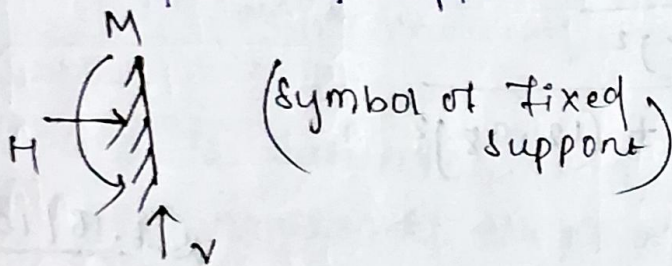
② Hinge Support →

The support which restricts both horizontal and vertical motion is called hinge support.



③ Fixed Support →

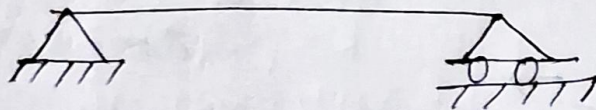
The support which restricts all the three motions (i.e. horizontal, vertical ^{and} bending) is called fixed support.



Q. Types of Beam

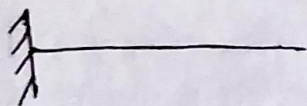
Dt. 17/10/23

① Simply Supported Beam →



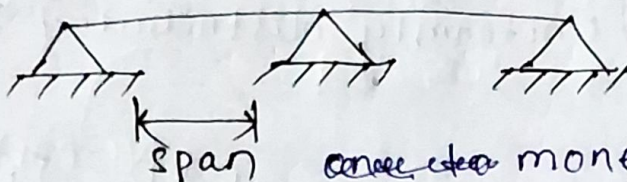
A beam having hinge support at one end and roller support at other end is known as a simply supported beam.

② Cantilever Beam →



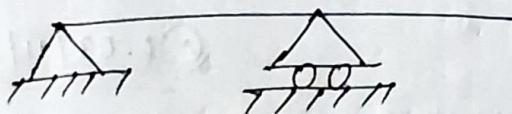
A beam having fixed support at one end and free at the other end is known as a cantilever beam.

③ Continuous Beam →

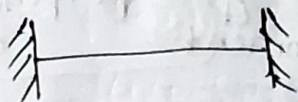


~~one or~~ more than one span present then i.e. called Continuous beam ..

④ Overhanging Beam →

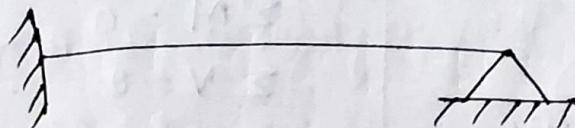


⑤ Fixed Beam →



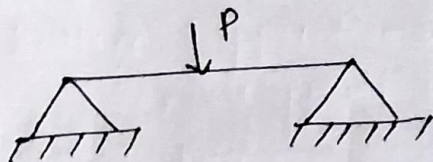
fixed support at both hand i.e. called fixed beam.

⑥ Propped Cantilever Beam →



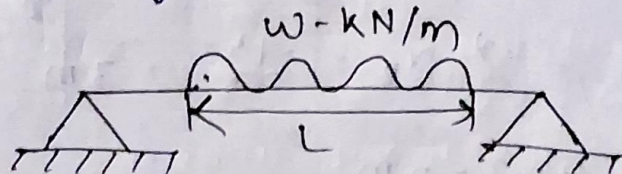
⑦ Types of Load

① Point Load →

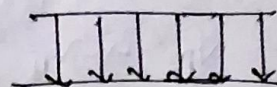


The load which acts at a single point is known as point load.

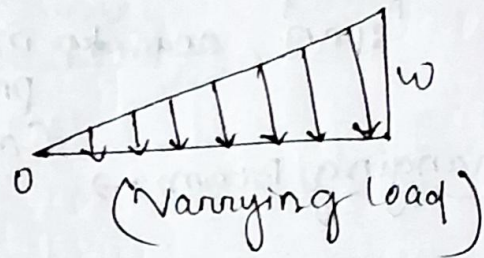
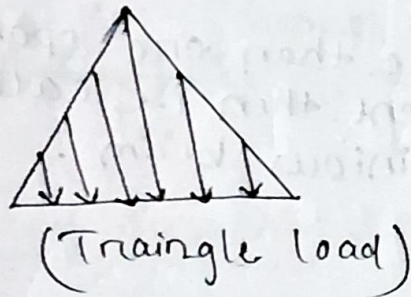
② Uniformly distributed load → (UDL)



Or



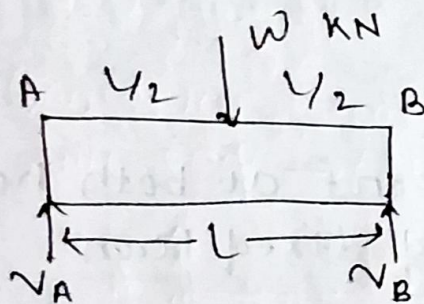
The load which acts over a length having uniform intensity through out the length is known as Uniformly distributed load.



$\begin{matrix} L & R \\ (+) & (-) \end{matrix}$

24.03/11/23

★



$$M_A = 0$$

$M = \text{Force} \times \text{perpendicular distance}$
moment

$$M_A = 0$$

$$N_B \times L = W \times \frac{L}{2}$$

$$\Rightarrow N_B = \frac{W \times \frac{L}{2}}{L} = W/2$$

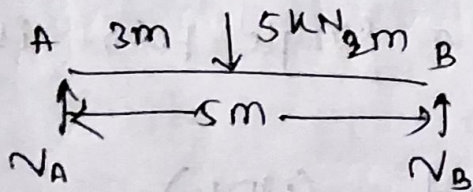
$$N_A + N_B = W$$

$$N_A = W - N_B$$

$$= W - W/2 = \frac{W}{2}$$

A body when equilibrium then
 $\Sigma M = 0$
 $\Sigma V = 0$
 $\Sigma H = 0$

★



$$M_A = 0$$

$$N_B \times L = W \times \frac{L}{2}$$

$$\Rightarrow N_B \times 5 = 5 \times 3$$

$$\Rightarrow N_B = \frac{15}{5} = 3 \text{ m}$$

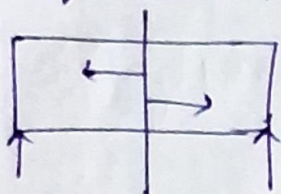
$$N_A + N_B = W$$

$$\Rightarrow N_A = W - N_B \Rightarrow 5 - 3 = 2 \text{ m}$$

Shear Force and Bending Moment $\Rightarrow$ 1.04/11/23

Shear Force $\Rightarrow$

The unbalanced vertical forces on a section towards the right or left of the section is called Shear Force.

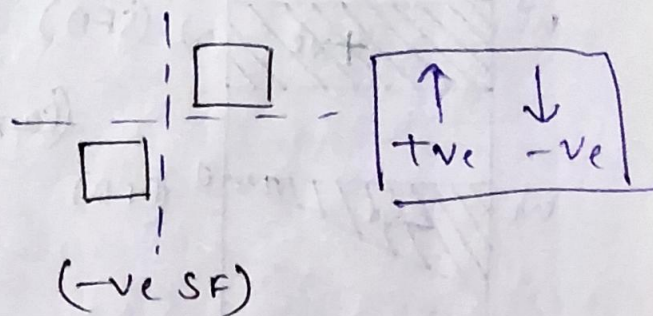
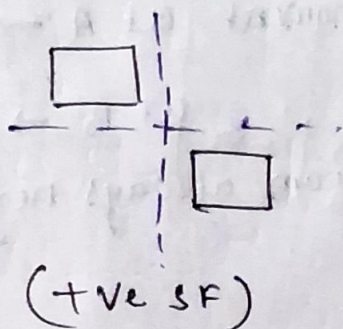


Sign convention ✓

$$\begin{array}{cc} \text{L} & \text{R} \\ + & - \end{array} \quad \begin{array}{cc} \text{L} & \text{R} \\ - & + \end{array}$$

When the shear force tries to lift the left hand portion upwards and right hand portion downwards it is called positive shear force.

When the shear force tries to lift the left hand portion downwards and right hand portion upwards it is called negative shear force.

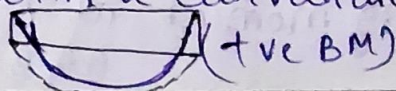


Bending Moment $\Rightarrow$

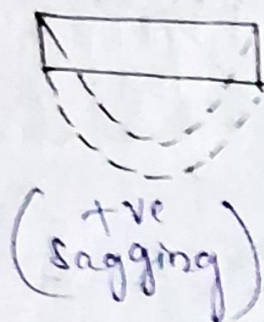
The algebraic sum of moment of all the forces either from left or right of the section, is called bending moment.

Sign convention ✓

The bending moment is taken as positive when it bends the beam with a curvature having concavity at the top.



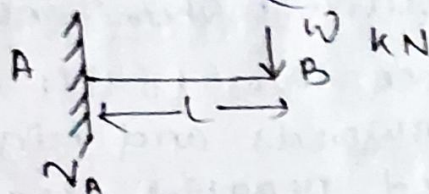
The bending moment is taken as negative when it bends the beam with a curvature having convexity at the top.



(-ve Hogging)

(-ve BM)

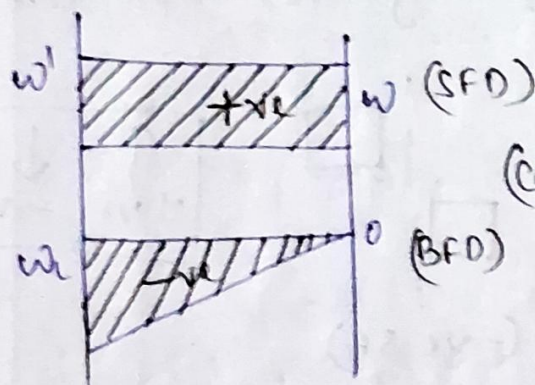
Cantilever Beam Subjected to point load at Free end $\rightarrow$ (Start from left) $\boxed{\uparrow (+) \downarrow (-)}$



$$N_A = W \text{ kN}$$

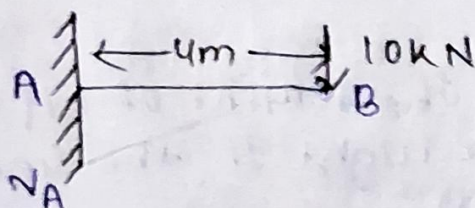
Shear Force at A $\rightarrow W \text{ kN}$
at B $\rightarrow W \text{ kN}$

Bending moment at A $\rightarrow W \times L$
kN-m
at B $\rightarrow 0$



(Cantilever beam always negative)

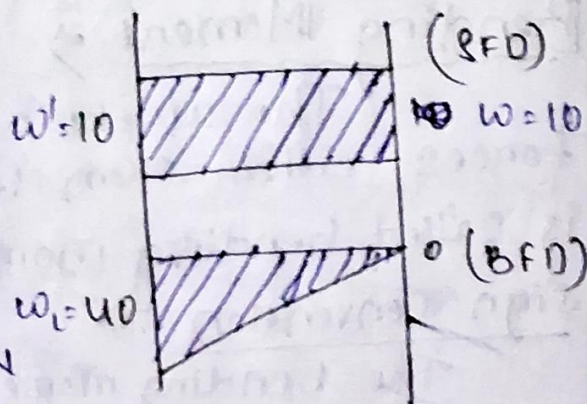
(Q)



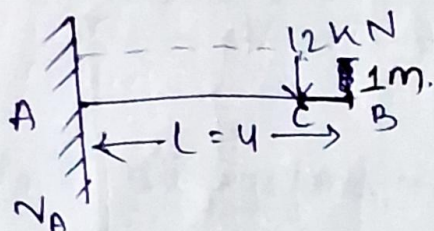
$$N_A = W \text{ kN}$$

Shear Force at A $\rightarrow 10 \text{ kN}$
at B $\rightarrow 10 \text{ kN}$

Bending moment at A $\rightarrow 10 \times 4 = 40 \text{ kNm}$
at B $\rightarrow 0$



(Q) Draw the SFD and BMD for a cantilever beam subjected to of length 4m. subjected to a point load of 12 kN at a distance of 1m. from the free end?



$V_A = W \text{ kN} = 12 \text{ kN}$
Shear force at A $\rightarrow W \text{ kN}$
 $= 12 \text{ kN}$

at B $\rightarrow 0 \text{ kN}$

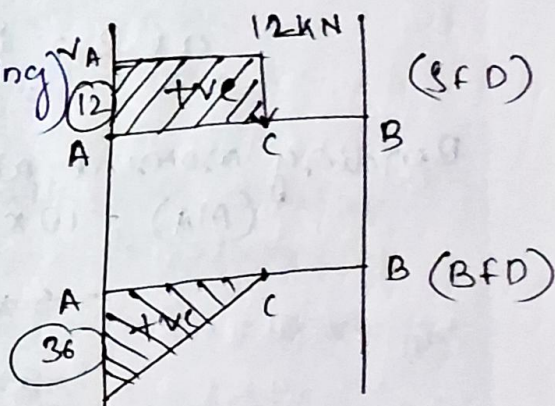
Bending moment at A $\rightarrow$
 ~~$W \times L \text{ kN.m}$~~

at C $\rightarrow 12 - 12 = 0$

Bending moment
at A $\rightarrow W L \text{ kNm}$ (Force acting from A - C)
(M_A) $= 12 \times 4 = 48 \text{ kNm}$
 ~~$= 36 \text{ kNm}$~~

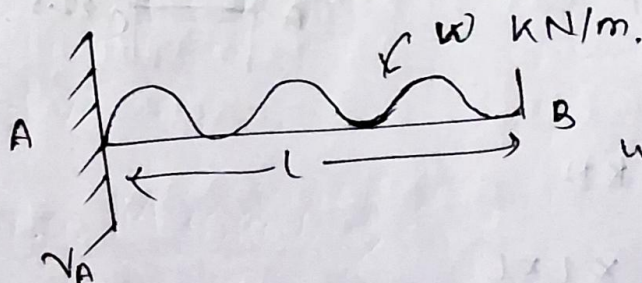
at B $\rightarrow 0$ (M_B)

at C $\rightarrow 0$ (M_C)



① L. 06/11/23

Cantilever Beam Subjected to UDL $\Rightarrow$ $\boxed{\downarrow (+) \uparrow (-)}$
(Start from right)



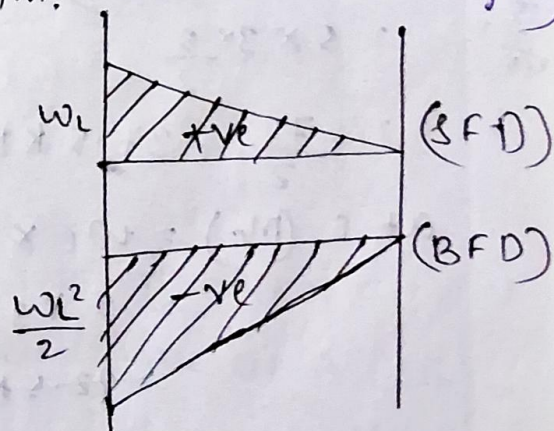
$V_A = W L \text{ kN}$

Shear force at A $= W L \text{ kN}$

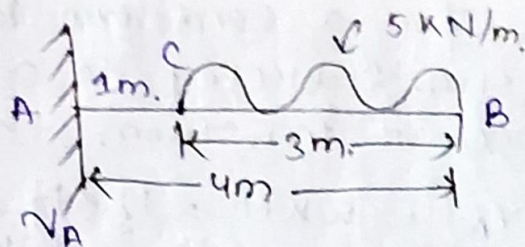
at B $= W L - W L$
 $= 0 \text{ kN}$

Bending moment at A (M_A) $= W \times L \times \frac{L}{2}$
 $= \frac{W L^2}{2}$

at B (M_B) $= 0$



(Q)



Draw the SFD and BFD?

$$V_A = 50 \text{ L kN}$$

$$= 5 \times 3 = 15 \text{ kN}$$

$$\text{Shear Force at A} = 15 \text{ kN}$$

$$\text{at C} = 15 \text{ kN}$$

$$= 5 \times 3 = 15 \text{ kN}$$

$$\text{at B} = 15 - 5 \times 3$$

$$= 15 - 15 = 0 \text{ kN}$$

Bending moment at A =

$$(M_A) = W \times L \times \frac{L}{2}$$

$$= 5 \times 3 \times \frac{3}{2}$$

$$= 5 \times 4 \times 2$$

$$= 5 \times 3 \times \left(\frac{3}{2} + 1\right)$$

$$= 5 \times 3 \times \left(\frac{3+2}{2}\right)$$

$$= 5 \times 3 \times \frac{5}{2}$$

$$= \frac{75}{2} = 37.5 \text{ kN}$$

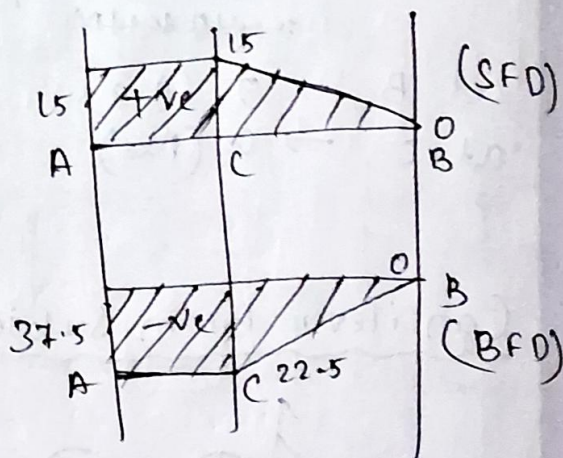
$$\text{at C } (M_C) = W \times L \times \frac{L}{2}$$

$$= 5 \times 3 \times \frac{3}{2}$$

$$= 22.5 \text{ kN}$$

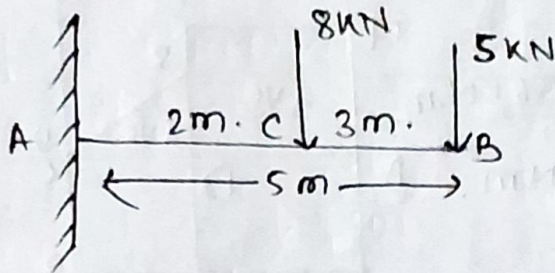
$$= \frac{45}{2} = 22.5 \text{ kN}$$

$$\text{at B } (M_B) = 0 \text{ (Free end)}$$



Q2) Draw the SFD and BMD?

01.07/11/23



(Cantilever beam subjected to point load at free end)

$$\begin{aligned} \uparrow V_A &= \text{net upward force} + \text{net downward force} \\ &= 8 + 5 = 13 \text{ kN} \end{aligned}$$

$$\text{Shear force at A (SF}_A) = 8 + 5 = 13 \text{ kN}$$

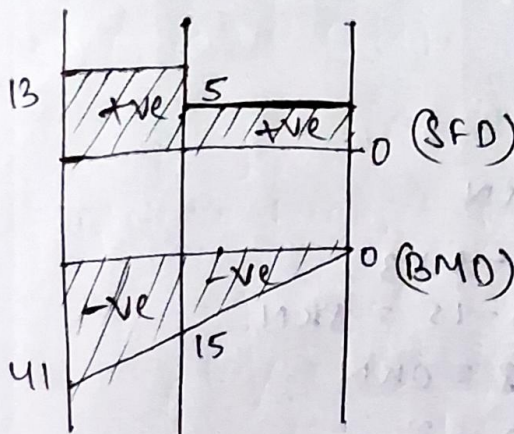
$$\text{at C (SF}_C) = 13 - 8 = 5 \text{ kN}$$

$$\text{at B (SF}_B) = 5 - 5 = 0 \text{ kN}$$

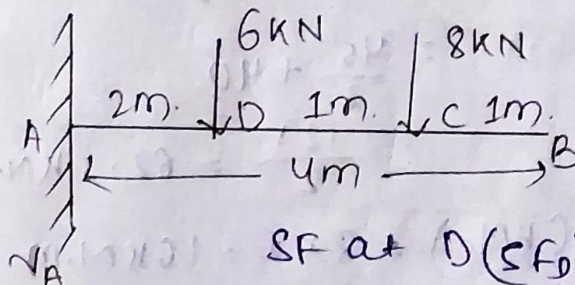
$$\begin{aligned} \text{Bending moment at A (M}_A) &= 8 \times 2 + 5 \times 5 \\ &= 41 \text{ kN}\cdot\text{m} \end{aligned}$$

$$\text{at C (M}_C) = 5 \times 3 = 15 \text{ kN}\cdot\text{m}$$

$$\text{at B (M}_B) = 0 \text{ (free end)}$$



Q3)



$$\uparrow V_A = 6 + 8 = 14 \text{ kN}$$

$$\text{Shear force at A (SF}_A) = 14 \text{ kN}$$

$$\text{SF at D (SF}_D) \rightarrow 14 - 6 = 8 \text{ kN}$$

$$\text{SF at C (SF}_C) \rightarrow 8 - 8 = 0 \text{ kN}$$

$$\text{SF at B (SF}_B) \rightarrow 0 \text{ kN}$$

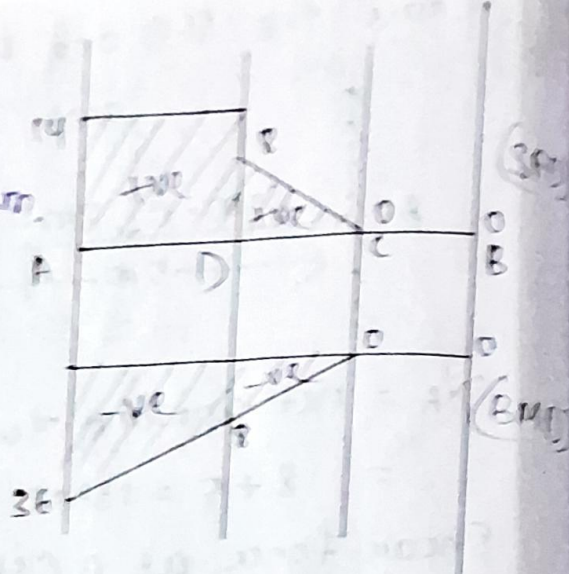
Bending moment

$$\begin{aligned}\text{at A (BM}_A\text{)} &= 6 \times 2 + 8 \times 3 \\ &= 12 + 24 = 36 \text{ kNm.}\end{aligned}$$

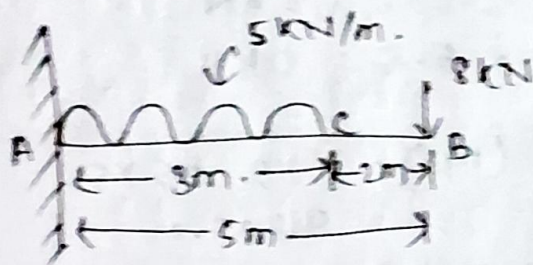
$$\text{at D (BM}_D\text{)} = 8 \times 1 = 8 \text{ kNm.}$$

$$\text{at C (BM}_C\text{)} = 0 \text{ kN}$$

$$\text{at B (BM}_B\text{)} = 0 \text{ kN}$$



(Q) cantilever beam subjected to UDL.



$$\begin{aligned}V_A &= 60 \text{ kN} \\ &= 5 \times 3 + 8 = 23 \text{ kN}\end{aligned}$$

$$\text{Shear Force at A (SF}_A\text{)} = 23 \text{ kN}$$

$$\begin{aligned}\text{at C (SF}_C\text{)} &= 23 - 5 \times 3 \\ &= 23 - 15 = 8 \text{ kN}\end{aligned}$$

$$\text{at B (SF}_B\text{)} = 8 - 8 = 0 \text{ kN}$$

$$\text{Bending moment at A (BM}_A\text{)} = 5 \times 3 \times \frac{3}{2} + 8 \times 5$$

$$= \frac{45}{2} + 40$$

$$= \frac{125}{2} = 62.5 \text{ kNm.}$$

$$\text{at C (BM}_C\text{)} = 8 \times 2 = 16 \text{ kNm.}$$

$$\text{at B (BM}_B\text{)} = 0 \text{ kN}$$

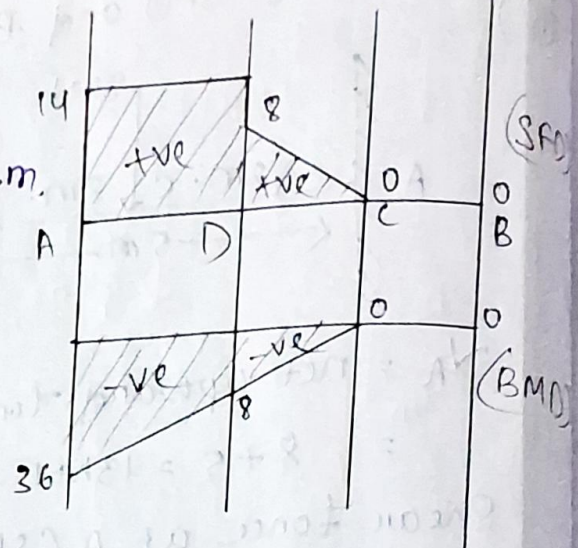
Bending moment

$$\text{at A (BM}_A\text{)} = 6 \times 2 + 8 \times 3 \\ = 12 + 24 = 36 \text{ kN.m.}$$

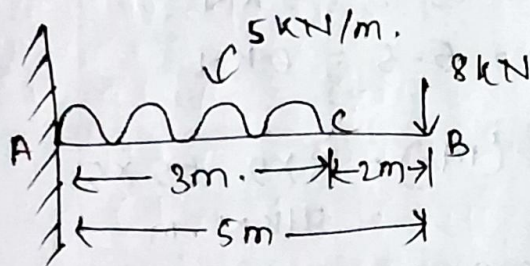
$$\text{at D (BM}_D\text{)} = 8 \times 1 = 8 \text{ kN.m.}$$

$$\text{at C (BM}_C\text{)} = 0 \text{ kN}$$

$$\text{at B (BM}_B\text{)} = 0 \text{ kN}$$



(Q) cantilever beam subjected to UDL.



$$V_A = WL \text{ kN} \\ = 5 \times 3 + 8 = 23 \text{ kN}$$

$$\text{Shear Force at A (SFA)} = 23 \text{ kN}$$

$$\text{at C (SFC)} = 23 - 5 \times 3 \\ = 23 - 15 = 8 \text{ kN}$$

$$\text{at B (SFB)} = 8 - 8 = 0 \text{ kN}$$

$$\text{Bending moment at A (BM}_A\text{)} = 5 \times 3 \times \frac{3}{2} + 8 \times 5$$

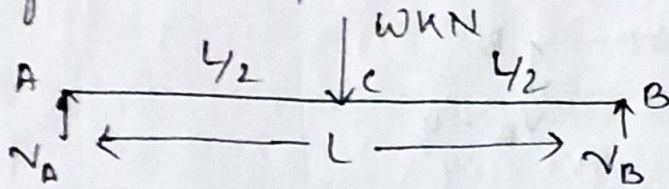
$$= \frac{45}{2} + 40$$

$$= \frac{125}{2} = 62.5 \text{ kN.m.}$$

$$\text{at C (BM}_C\text{)} = 8 \times 2 = 16 \text{ kN.m}$$

$$\text{at B (BM}_B\text{)} = 0 \text{ kN}$$

Simply Supported Beam subjected to point load $\rightarrow$



$$M_A = 0$$

$$V_B \times L = W \times \frac{L}{2}$$

$$\Rightarrow V_B = \frac{W \times \frac{L}{2}}{L}$$

$$\Rightarrow \boxed{V_B = \frac{W}{2}}$$

$V_A, V_B \rightarrow$ upward force
 $W \rightarrow$ downward force

$$V_A + V_B = W$$

$$\Rightarrow V_A = W - V_B$$

$$= W - \frac{W}{2}$$

$$\boxed{V_A = \frac{W}{2}}$$

$$\boxed{\star \text{ Bending moment at centre } = \frac{WL}{4}}$$

$$\text{Shear force at A } \boxed{(SFA) = \frac{W}{2}}$$

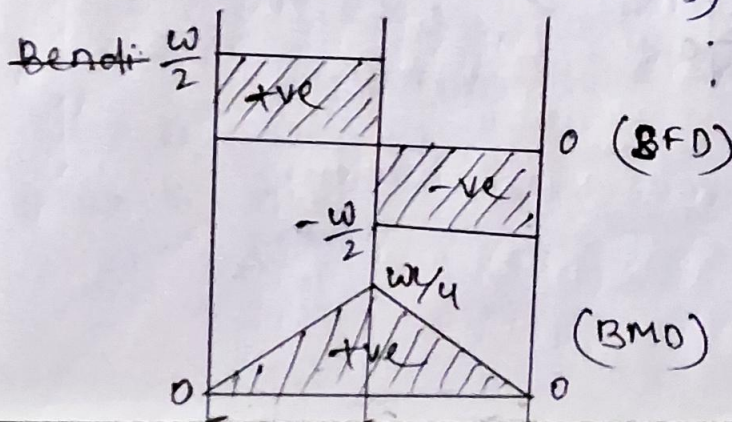
$$\text{at C } (SFC) = \frac{W}{2} - W = -\frac{W}{2}$$

$$\text{at B } (SFB) = -\frac{W}{2} + \frac{W}{2} = 0$$

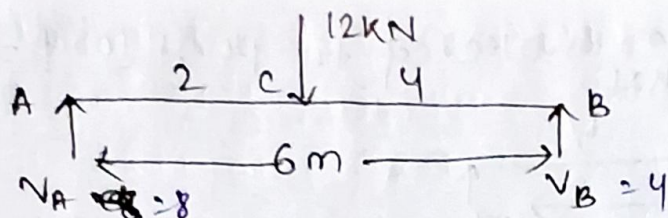
$$\text{Bending moment at A } (BMA) = 0 \text{ (free end)}$$

$$\text{at C } (BMC) = \frac{W}{2} \times \frac{L}{2} = \frac{WL}{4}$$

$$\text{at B } (BMB) = 0 \text{ (free end)}$$



(Q)



$$M_A = 0$$

$$V_B \times 6 = 12 \times 2$$

$$\Rightarrow V_B = \frac{24}{6} = 4 \text{ kN}$$

$$V_A + V_B = W$$

$$\Rightarrow V_A + 4 = 12$$

$$\Rightarrow V_A = 12 - 4 = 8 \text{ kN}$$

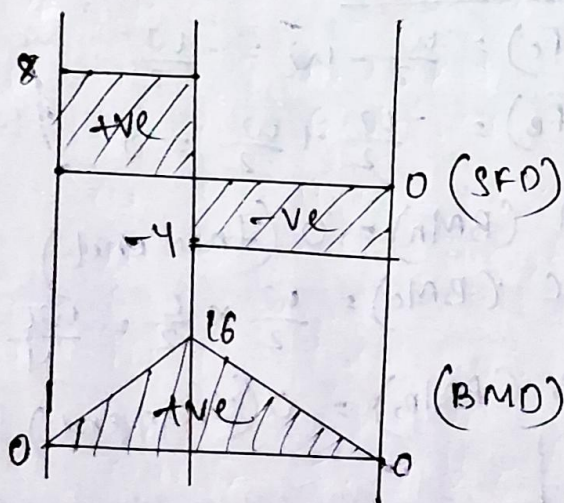
$$\text{SF at A} = 8 \text{ kN}$$

$$\text{at C} = 8 - 12 = -4 \text{ kN}$$

$$\text{at B} = -4 + 4 = 0 \quad (\because V_B = 4)$$

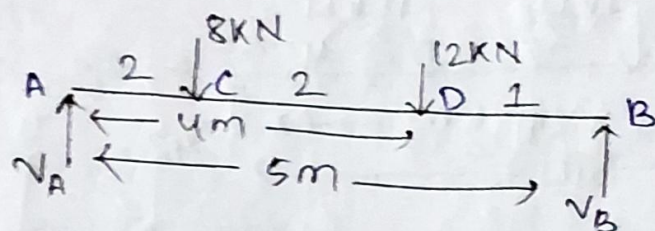
$$\text{BM at A} = 0 \text{ kN}$$

$$\text{at C} = 4 \times 4 = 16 \text{ kN.m} \quad (\because V_B \times L)$$



16) Draw the SFD and BMD for a simply supported beam of length 5m. carrying two point loads of 8kN and 12kN at a distance of 2m and 4m from the left end.

21-08/11/23



$$M_A = 0$$

$$V_B \times 5 = 8 \times 2 + 12 \times 4$$

$$\rightarrow V_B \times 5 = 64$$

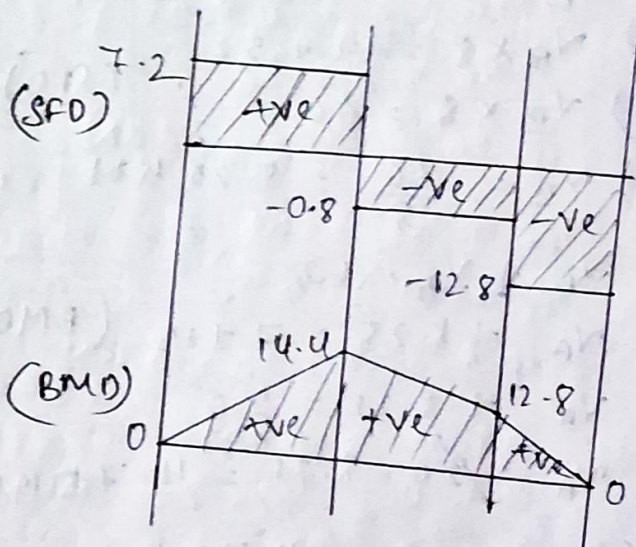
$$\rightarrow V_B = 64/5 = 12.8 \text{ kN}$$

$$V_A + V_B = W$$

$$\rightarrow V_A + 12.8 = 8 + 12$$

$$\rightarrow V_A + 12.8 = 20$$

$$\rightarrow V_A = 20 - 12.8 = 7.2 \text{ kN}$$



$$\text{SF at A} = 7.2 \text{ kN}$$

$$\text{at C} = 7.2 - 8 = -0.8 \text{ kN}$$

$$\text{at D} = -0.8 - 12 = -12.8 \text{ kN}$$

$$\text{at B} = -12.8 + 12.8 = 0$$

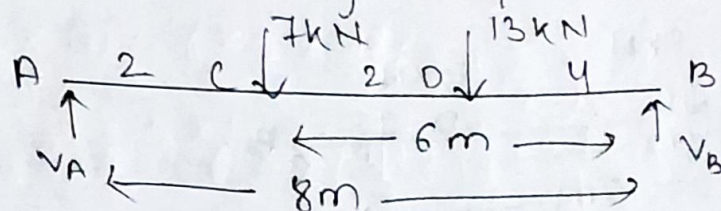
$$\text{BM at A} = 0 \text{ kN}$$

$$\begin{aligned} \text{at C} &= 12.8 \times 3 - 12 \times 2 \\ &= 14.4 \text{ kN.m} \end{aligned}$$

$$\text{at D} = 12.8 \times 1 = 12.8 \text{ kN.m}$$

$$\text{at B} = 0 \text{ kN}$$

(Q) Draw the SFD and BMD for a simply supported beam of length 8m carrying two point loads of 7kN and 13kN at a distance of 6m and 4m from the right end.



$$M_A = 0$$

$$V_B \times 8 = 7 \times 2 + 13 \times 4$$

$$\Rightarrow V_B \times 8 = 14 + 52$$

$$\Rightarrow V_B \times 8 = 66$$

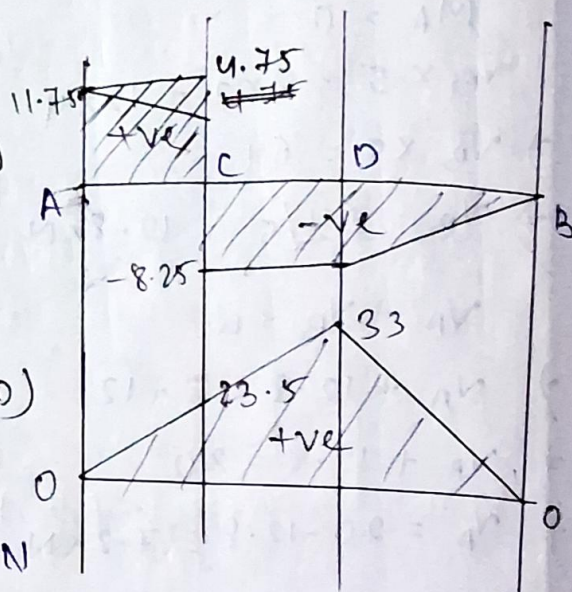
$$\Rightarrow V_B = \frac{66}{8} = 8.25 \text{ kN}$$

$$V_A + V_B = W$$

$$\Rightarrow V_A + 8.25 = 7 + 13 \quad (\text{BMD})$$

$$\Rightarrow V_A + 8.25 = 20$$

$$\Rightarrow V_A = 20 - 8.25 = 11.75 \text{ kN}$$



$$\text{SF at A} = 11.75 \text{ kN}$$

$$\text{at C} = 11.75 - 7 = 4.75 \text{ kN}$$

$$\text{at D} = 4.75 - 13 = -8.25 \text{ kN}$$

$$\text{at B} = -8.25 + 8.25 = 0 \text{ kN}$$

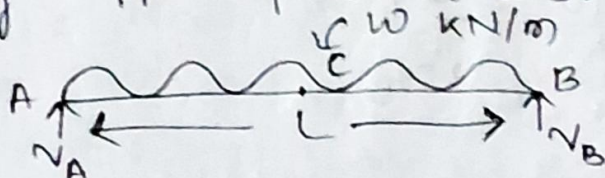
$$\text{BM at A} = 0 \text{ kN.m.}$$

$$\text{at B} = 0 \text{ kN.m.}$$

$$\begin{aligned} \text{at C} &= 8.25 \times 6 - 13 \times 2 \\ &= 23.5 \text{ kN.m.} \end{aligned}$$

$$\text{at D} = 8.25 \times 4 = 33 \text{ kN.m.}$$

2) Simply Supported Beam Subjected to UDL $\rightarrow$



$$M_A = 0$$

$$\Rightarrow N_B \times L = w \times L \times \frac{L}{2}$$

$$\Rightarrow N_B = \frac{wL}{2}$$

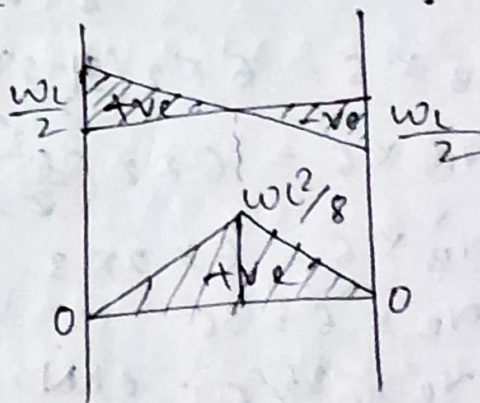
$$N_A + N_B = w \times L$$

$$\Rightarrow N_A = wL - N_B$$

$$\Rightarrow N_A = wL - \frac{wL}{2}$$

$$= \frac{2wL - wL}{2}$$

$$= \frac{wL}{2}$$



Shear force at $A = \frac{wL}{2}$

at $B = \frac{wL}{2} - wL = -\frac{wL}{2}$

Bending moment at $A = 0$ (M_A)

at $B = 0$ (M_B)

moment at center

$$M_C = \frac{wL}{2} \times \frac{L}{2} - w \times \frac{L}{2} \times \frac{L}{4}$$

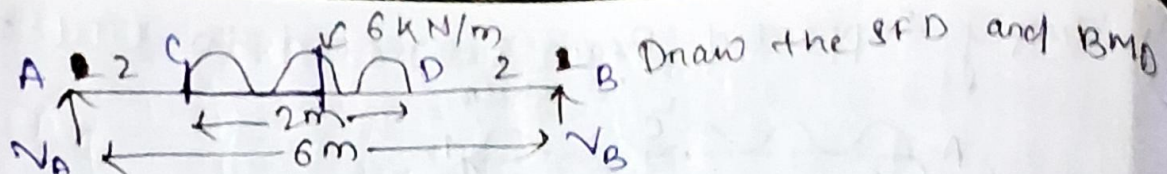
$$= \frac{wL^2}{4} - \frac{wL^2}{8}$$

$$= \frac{2wL^2 - wL^2}{8}$$

$$M_C = \frac{wL^2}{8} \quad \text{IMR}$$

(bending moment of the simply supported beam carrying an UDL)

(Q) ①



$$M_A = 0$$

$$\rightarrow N_B \times 6 = 6 \times 2 \times \left(2 + \frac{2}{2}\right)$$

$$\rightarrow N_B \times 6 = 6 \times 2 \left(\frac{2+2}{2}\right)$$

$$\rightarrow N_B \times 6 = 6 \times 2 \left(\frac{6}{2}\right)$$

$$\rightarrow N_B \times 6 = 6 \times 2 \times 3 \quad (\text{SFD})$$

$$\rightarrow N_B \times 6 = 36$$

$$\rightarrow N_B = 36/6 = 6 \text{ kN}$$

$$N_A + N_B = W$$

$$\rightarrow N_A + 6 = 6 \times 2$$

$$\rightarrow N_A = 12 - 6 = 6 \text{ kN}$$

$$\text{SF at A} = 6 \text{ kN}$$

$$\text{at C} = 6 - 2 = 4 \text{ kN}$$

$$\text{at D} = 6 - 6 \times 2$$

$$= 6 - 12$$

$$= -6 \text{ kN}$$

$$\text{at B} = -6 + 6 = 0 \text{ kN}$$

$$\text{BM at A} = 0$$

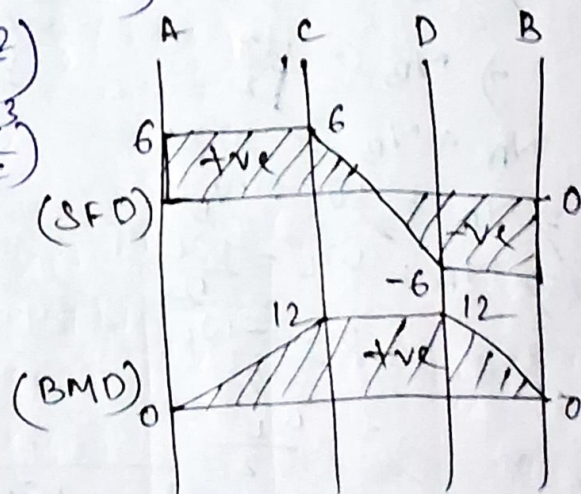
$$\text{at M}_B = 0$$

$$M_C = 6 \times 4 - 6 \times 2 \times \frac{2}{2}$$

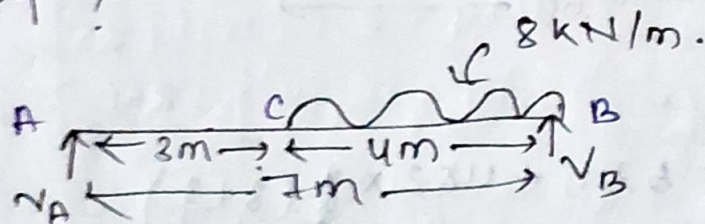
$$= 12 \text{ kN.m.}$$

$$M_D = N_B \times 2$$

$$= 6 \times 2 = 12 \text{ kN.m}$$



10. ② Draw the SFD and BMD for a simply supported beam of length 7m. carrying an UDL of 8 kN/m over a span of 4m. starting from right end? Dt. 10/11/23



$$M_A = 0$$

$$\rightarrow V_B \times 7 = 8 \times 4 \times \left(3 + \frac{4}{2}\right)$$

$$\rightarrow V_B \times 7 = 8 \times 4 \times 5$$

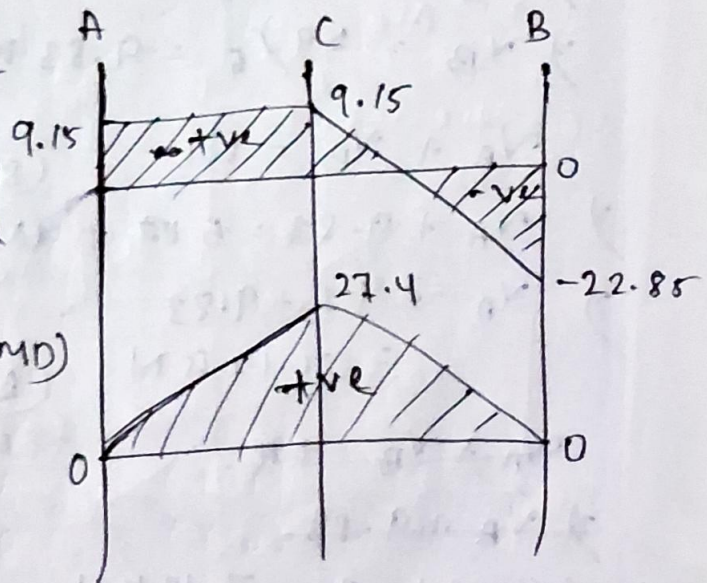
$$\rightarrow V_B \times 7 = 160 \quad (\text{SFD})$$

$$\rightarrow V_B = 160/7 = 22.85 \text{ kN}$$

$$V_A + V_B = W$$

$$\rightarrow V_A + 22.85 = 8 \times 4 \quad (\text{BMD})$$

$$\rightarrow V_A = 32 - 22.85 = 9.15 \text{ kN}$$



$$\text{SF at A} = 9.15 \text{ kN}$$

$$\text{at C} = 9.15 \text{ kN}$$

$$\text{at B} = 9.15 - 8 \times 4$$

$$= 9.15 - 32 = -22.85 \text{ kN}$$

$$= -22.85 + 22.85 = 0 \text{ kN}$$

$$\text{BM at A} = 0$$

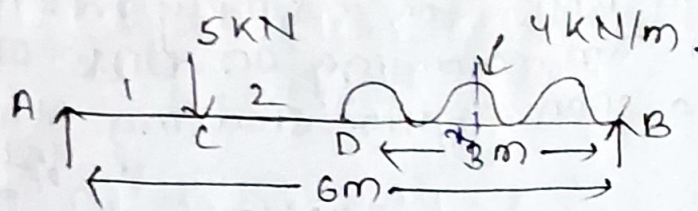
$$\text{at B} = 0$$

$$\text{BM}_C = 22.85 \times 4 - 8 \times 4 \times \frac{4}{2}$$

$$= 91.4 - 64$$

$$= 27.4 \text{ kN.m.}$$

(Q)



$$M_A = 0$$

$$2) \quad N_B \times 6 = 5 \times 1 + 4 \times 3 \times \frac{1}{2} \left(3 + \frac{3}{2}\right)$$

$$3) \quad N_B \times 6 = 5 + 12 \times 9/2$$

$$4) \quad N_B \times 6 = 59$$

$$5) \quad N_B = 59/6 = 9.83 \text{ kN}$$

$$N_A + N_B = WL \quad ; \quad (\text{SFD})$$

$$6) \quad N_A + 9.83 = 5 + 4 \times 3$$

$$7) \quad N_A = 17 - 9.83 = 7.17 \text{ kN} \quad (\text{BMD})$$

$$N_A + N_B = WL$$

$$8) \quad N_A + 9.83$$

$$\text{SF at A} = 7.17 \text{ kN}$$

$$\text{at C} = 7.17 - 5 = 2.17 \text{ kN}$$

$$\text{at D} = 2.17 \text{ kN}$$

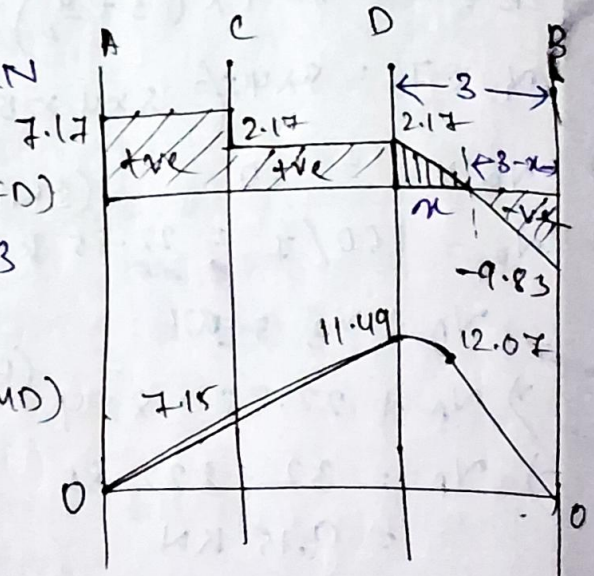
$$\text{at B} = 2.17 - 4 \times 3 = -9.83 \text{ kN}$$

$$= -9.83 + 9.83 = 0 \text{ kN}$$

$$\text{at BM}_A = 0$$

$$\text{BM}_B = 0$$

$$\text{BM}_C = 9.83 \times 5 - 4 \times 3 \times \left(2 + \frac{3}{2}\right) = 7.15 \text{ kN}\cdot\text{m}$$



$$BM_D = 9.83 \times 3 - 4 \times 3 \times \frac{3}{2}$$

$$= 11.49 \text{ kN.m.}$$

$$\frac{2.17}{x} = \frac{9.83}{3-x}$$

$$\Rightarrow 2.17(3-x) = 9.83x$$

$$\Rightarrow 6.51 - 2.17x = 9.83x$$

$$\Rightarrow 6.51 = 9.83x + 2.17x = 12x$$

$$\Rightarrow 12x = 6.51$$

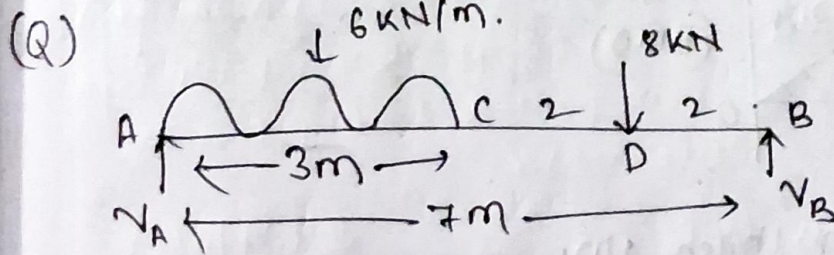
$$\Rightarrow x = \frac{6.51}{12} = 0.54$$

$$\therefore 3-x = 3 - 0.54 = 2.46$$

$$M_{0.54} = 9.83 \times 2.46 - 4 \times 2.46 \times \frac{2.46}{2}$$

$$= 12.07 \text{ kN.m.}$$

★ In a section when SF zero that point Bending moment will be maxm



$$M_A = 0$$

$$\Rightarrow N_B \times 7 = 6 \times 3 \times \left(4 + \frac{3}{2}\right) + 8 \times 5$$

$$\Rightarrow N_B \times 7 = 12 \times 11\frac{1}{2} + 40$$

$$\Rightarrow N_B \times 7 = 66 + 40$$

$$\Rightarrow N_B = \frac{106}{7} = 15.14 \text{ kN}$$

$$N_A + N_B = WL$$

$$\Rightarrow N_A + 15.14 = 6 \times 3 + 8$$

$$\Rightarrow N_A + 15.14 = 26$$

$$\Rightarrow N_A = 26 - 15.14 = 10.86 \text{ kN}$$

$$SF \text{ at } A = 10.86 \text{ kN}$$

$$\text{at } C = 10.86 - 6 \times 3$$

$$= -7.14 \text{ kN}$$

$$\text{at } D = -7.14 - 8 = -15.14 \text{ kN}$$

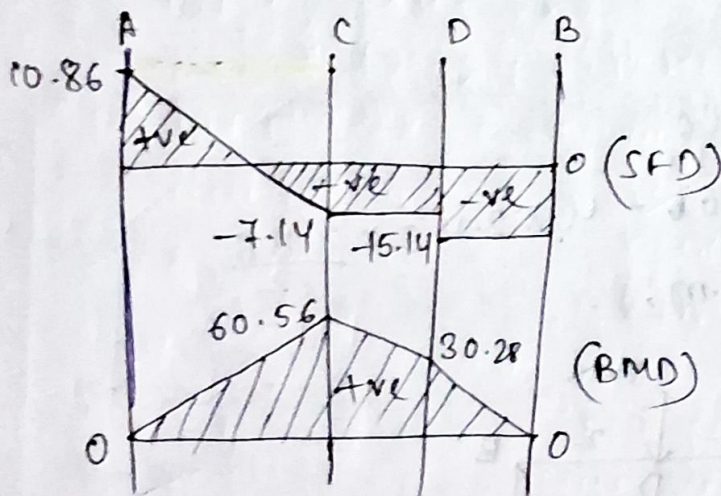
$$\text{at } B = -15.14 + 15.14 = 0 \text{ kN}$$

$$\text{At } M_A = 0$$

$$M_C = 15.14 \times 4 = 60.56 \text{ kN}\cdot\text{m}$$

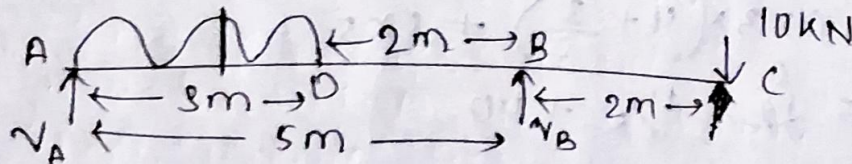
$$M_D = 15.14 \times 2 = 30.28 \text{ kN}\cdot\text{m}$$

$$M_B = 0$$



(Q) over hanging beam under UDL.

↓ 5 kN/m.



$$M_A = 0$$

$$\Rightarrow V_B \times 5 = 5 \times 3 \times \frac{3}{2} + 10 \times 7$$

$$\Rightarrow V_B \times 5 = 92.5$$

$$\Rightarrow V_B = \frac{92.5}{5} = 18.5 \text{ kN}$$

$$N_A + N_B = W_L$$

$$N_A + 18.5 = 5 \times 3 + 10$$

$$N_A + 18.5 = 15 + 10$$

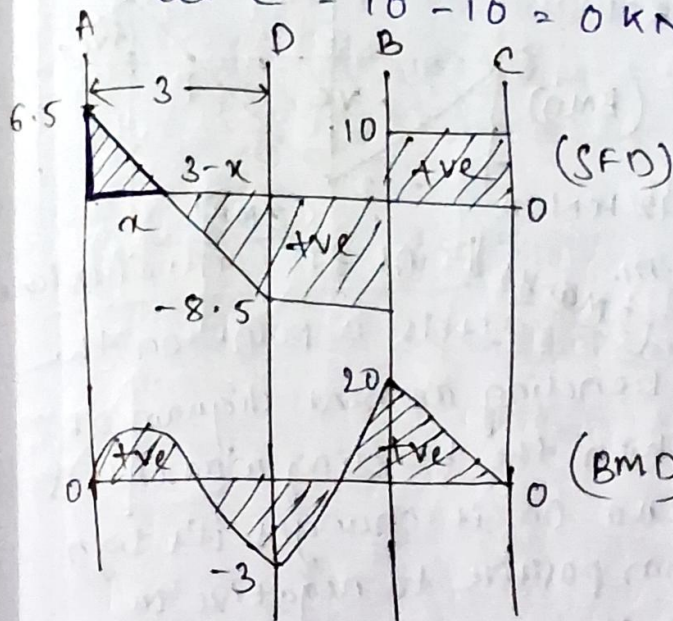
$$N_A = 6.5 \text{ kN}$$

$$\text{SF at A} = 6.5 \text{ kN}$$

$$\text{at D} = 6.5 - 5 \times 3 = -8.5 \text{ kN}$$

$$\text{at B} = -8.5 + 18.5 = 10 \text{ kN}$$

$$\text{at C} = 10 - 10 = 0 \text{ kN}$$



$$\text{BM at A} = 0$$

$$\text{at C} = 0$$

$$\text{at D} = 18.5 \times 2$$

$$-10 \times 4 = -3$$

$$\text{KN.m.}$$

$$\text{at B} = 10 \times 2 = 20 \text{ KN.m.}$$

$$\frac{6.5}{x} = \frac{8.5}{3-x}$$

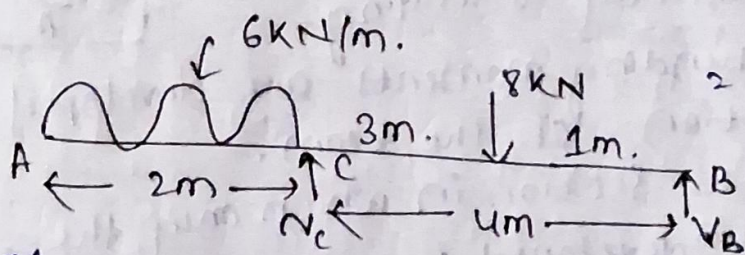
$$\Rightarrow 6.5(3-x) = 8.5x$$

$$M_{1.7} = 18.5 \times (2 + 1.7) - 5 \times 1.7 \times \frac{1.7}{2}$$

$$-10(4 + 1.7)$$

$$= 4.22 \text{ KN.m}$$

(Q)



$$M_B = 0$$

$$\Rightarrow N_C \times 4 = 8 \times 1 + 6 \times 2 \times \left(\frac{2}{2} + 4\right)$$

$$\Rightarrow N_C \times 4 = 8 + 60$$

$$\Rightarrow N_C = \frac{68}{4} = 17$$

$$V_C + V_B = 6 \times 2 + 8$$

$$V_B = 20 - 17 = 3 \text{ kN}$$

$$\text{SF at A} = 0 \text{ kN}$$

$$\begin{aligned} \text{at C} &= \cancel{6 \times 2} \quad (\text{SFD}) \\ -6 \times 2 &= -12 \text{ kN} \\ -12 + 17 &= 5 \text{ kN} \\ &= \cancel{-12 + 17} = 5 \text{ kN} \end{aligned}$$

$$\text{at D} = 5 - 8 = -3 \text{ kN}$$

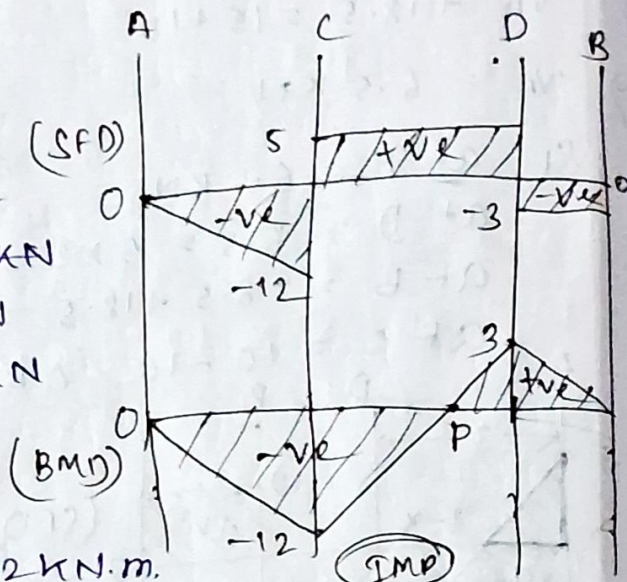
$$\text{at B} = -3 + 3 = 0 \text{ kN}$$

$$\text{BM at A} = 0$$

$$\begin{aligned} \text{at C} &= 3 \times 4 - 8 \times 3 \\ &= 12 - 24 = -12 \text{ kN.m.} \end{aligned}$$

$$\text{at D} = 3 \times 1 = 3 \text{ kN.m.}$$

$$\text{at B} = 0$$



12 marks Point of Contraflexure

It is a point on the bending moment diagram at which the bending moment is zero on it changes its sign from positive to negative or vice-versa.

Bending Stress in Beam →

Q.14/11/23

→ When a beam is subjected to loading shear force and bending moments are developed throughout the section of the beam.

$$\sigma = \frac{P}{A}$$

But there will be section in which only the bending moment is developed but no shear force, this type of bending moment is called simple bending or pure bending and the stress developed due to this bending is called bending stress.

$$\text{Shear force} = 0$$

$$\text{bending moment} = \text{max}$$

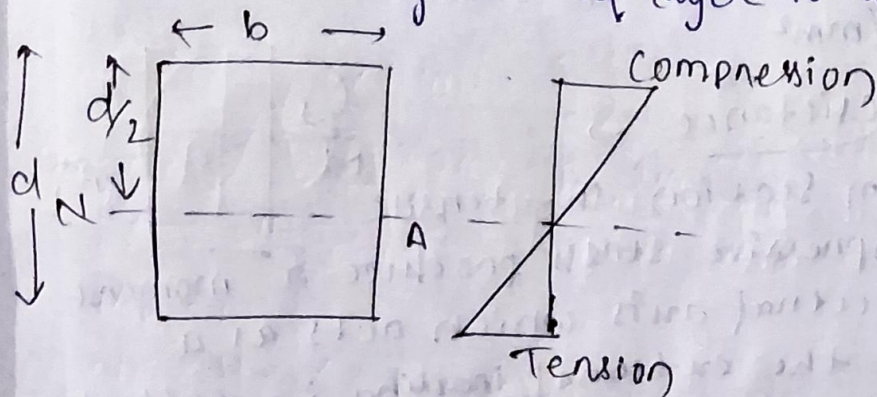
Theory of simple bending →

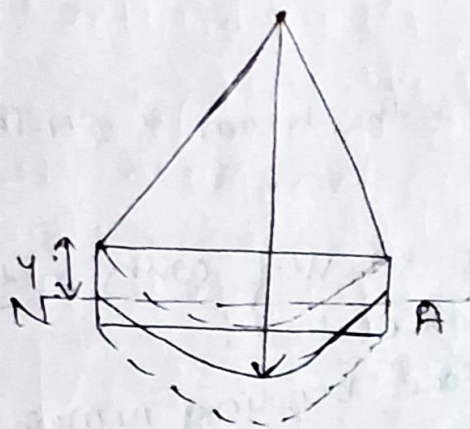
- Assumptions
- ① The material of the beam is perfectly homogenous and isotropic.
 - ② The material of the beam is stressed within the elastic limit and obeys Hooke's law.
 - ③ The section which is plane before bending remains plane after bending.
 - ④ Each layer of the beam is free to expand or contract independently.
 - ⑤ The value of modulus of elasticity is same in tension and compression.
 - ⑥ The beam is in equilibrium.

Theory of simple bending →

Dt. 16/11/23

When a beam is subjected to bending it bends in such a way that 1 portion of the beam remains in compression and the other portion remains in tension and in between them there is a layer where the stress is zero that layer is known as neutral layer and the axis joining neutral layer is known as neutral axis.





σ_b = Bending stress

R = Radius of curvature

E = modulus of elasticity

y = Depth of NA

$$\frac{\sigma_b}{y} = \frac{E}{R}$$

$$\sigma_b = \frac{E}{R} \times y$$

- (Q) A steel wire of 5mm diameter is bent in to a circular shape of depth radius 5m. Determine the max^m stress develop in the wire if $E = 200 \text{ GPa}$.

Ans $E = 200 \text{ GPa} = 200 \times 10^3 \text{ MPa}$

$\phi R = 5 \text{ mm}$

$d = 5 \text{ m} = 5 \times 10^3 \text{ mm}$

$\sigma_b \text{ max}^m = ?$

So $y = \frac{5}{2} = 2.5 \text{ mm}$.

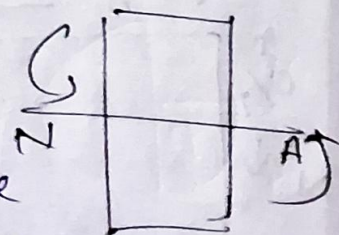
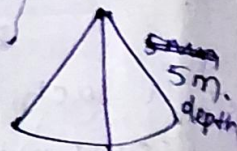
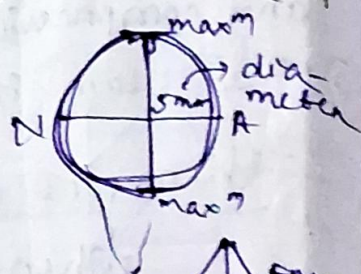
$$\sigma_b = \frac{E}{R} \times y$$

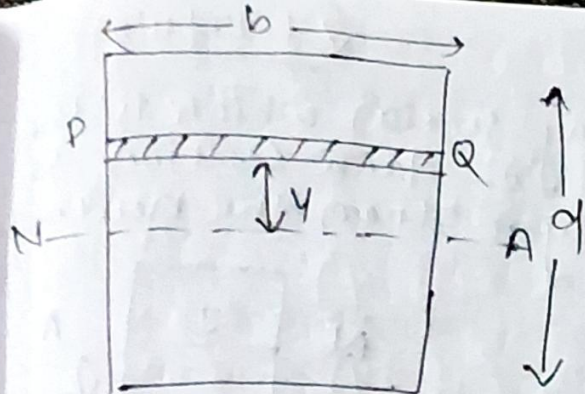
$$= \frac{200 \times 10^3}{5 \times 10^3} \times 2.5$$

$$= 100 \text{ N/mm}^2$$

Moment of Resistance →

In beam section the tensile stress and compressive stress produce a moment about the neutral axis which acts as a resistance to the external loading i.e called moment of resistance.





Equation Of Bending

Long question (10 Mark / 2 Mark)
Consider a small layer PQ of area δa .
 y = Distance of layer from NA.

$$\sigma = \frac{E}{R} \times y$$

$$\sigma = \frac{F}{A}$$

$$P = \sigma \cdot A$$

Total Force on layer PQ = $\sigma \cdot \delta a$

$$= \frac{E}{R} y \cdot \delta a$$

Moment produced by the layer PQ about NA.
= Force $\times$ distance

$$= \frac{E}{R} y \delta a \cdot y$$

$$= \frac{E}{R} y^2 \delta a$$

$M = \text{Force} \times \text{Perpendicular distance}$

Total moment produced about NA

$$= \int \frac{E}{R} y^2 \delta a$$

$$= \frac{E}{R} \int y^2 \delta a$$

$$M = \frac{E}{R} I$$

$$\Rightarrow \frac{M}{I} = \frac{E}{R}$$

Again we know that

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\text{So, } \frac{\sigma}{y} = \frac{E}{R} = \frac{M}{I}$$

This is known as bending eqⁿ.

Where,

σ = Bending stress

y = distance from NA

E = Modulus of elasticity

R = Radius of curvature

M = Bending moment

I = moment of Inertia

21.17/11/23

Section Modulus →

It is the ppt of any section which is the ratio of moment of inertia of given section to the distance of extreme layer from the neutral axis.

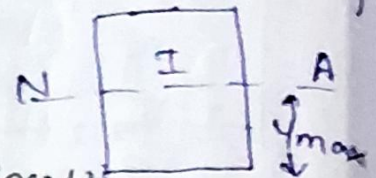
→ It is denoted by 'Z'.

$$Z = \frac{I}{Y_{max}}$$

Where, I = moment of inertia

Y_{max} = Distance of extreme layer from NA

Z = Section modulus



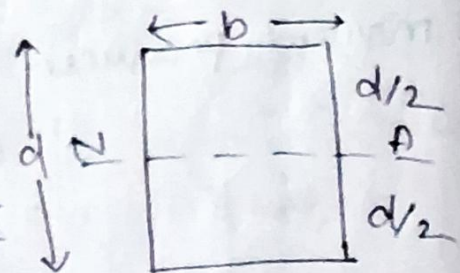
Rectangular Section →

* moment of inertia of rectangular section (I) = $\frac{bd^3}{12}$

$$Y_{max} = \frac{d}{2}$$

$$Z = \frac{I}{Y_{max}} = \frac{\frac{bd^3}{12}}{\frac{d}{2}} = \frac{bd^3}{12} \times \frac{2}{d} = \frac{bd^2}{6}$$

$$Z = \frac{bd^2}{6} \quad (\text{Section modulus in rectangular section})$$



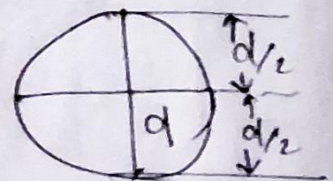
Circular Section →

* moment of inertia of circular section (I) = $\frac{\pi}{64} d^4$

$$Y_{max} = \frac{d}{2}$$

$$Z = \frac{I}{Y_{max}} = \frac{\frac{\pi}{64} d^4}{\frac{d}{2}} = \frac{\pi}{64} d^4 \times \frac{2}{d} = \frac{\pi}{32} d^3$$

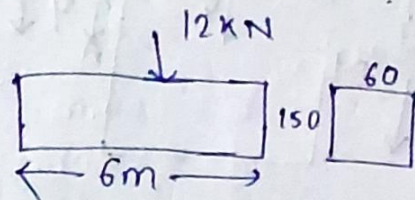
$$Z = \frac{\pi}{32} d^3 \quad (\text{Section modulus in circular section})$$



(Q) A rectangular beam of width 60mm and depth 150mm is simply supported over a span of 6m. If the beam is subjected to a central point load of 12kN. Find the max^m bending stress developed in the section?

A. Given,

width (b) = 60mm
depth (d) = 150mm



Simply supported beam

subjected to a point load at centre (M)

$$M = \frac{WL}{4} = \frac{12 \times 6}{4} = 18 \text{ kN.m} = 18 \times 10^6 \text{ N}$$

$$\sigma_{\max} = ?$$

$$\frac{\sigma}{y} = \frac{M}{I}$$

$$\sigma_{\max} = \frac{M}{I} \cdot y_{\max}$$

$$I = \frac{bd^3}{12} = \frac{60 \times 150^3}{12} = 16875000$$

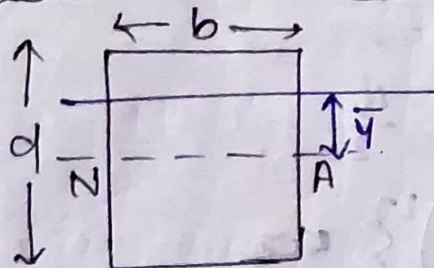
(moment of inertia of rectangular section)

$$y_{\max} = \frac{d}{2} = \frac{150}{2} = 75$$

$$\sigma_{\max} = \frac{18 \times 10^6}{16875000} \times 75 = 80 \text{ N/mm}^2 \quad \underline{\underline{\text{Ans}}}$$

Shear Stress in Beam $\Rightarrow$

Q1.20/14/23



F = Shear force in Beam

b = width of beam

I = moment of inertia

τ = Shear stress

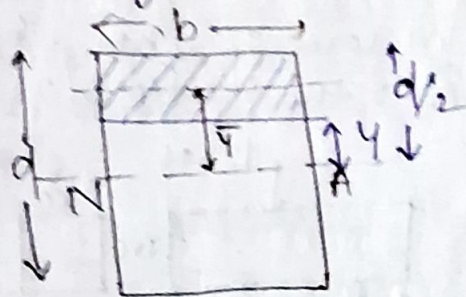
$\bar{y}$ = Distance of layer from the NA (Neutral Axis)

A = b x d (rectangular)

A = $\frac{\pi}{4} d^2$ (circular)

$$\tau = F \times \frac{A \bar{y}}{I b}$$

Q. Rectangular Section →



$$\tau = f \times \frac{A\bar{y}}{Ib}$$

$$A = b \times \left(\frac{d}{2} - y\right)$$

$$\bar{y} = y + \frac{\left(\frac{d}{2} - y\right)}{2}$$

$$= y + \frac{1}{2} \left(\frac{d}{2} - y\right)$$

$$= y + \frac{d}{4} - \frac{y}{2}$$

$$\bar{y} = \frac{d}{4} + \frac{y}{2}$$

$$\tau = f \times \frac{b \left(\frac{d}{2} - y\right) \times \left(\frac{d}{4} + \frac{y}{2}\right)}{I \times b}$$

$$= \frac{f}{I} \left(\frac{d}{2} - y\right) \times \frac{1}{2} \left(\frac{d}{2} + y\right)$$

$$= \frac{f}{2I} \left\{ \left(\frac{d}{2}\right)^2 - y^2 \right\}$$

$$\boxed{(a+b)(a-b) = a^2 - b^2}$$

$$= \frac{f}{2I} \left(\frac{d^2}{4} - y^2 \right)$$

$$\boxed{\tau = \frac{f}{2I} \left(\frac{d^2}{4} - y^2 \right)}$$

Min^m shear stress at $y = \frac{d}{2}$

$$\tau_{\min} = \frac{f}{2I} \left\{ \frac{d^2}{4} - \left(\frac{d}{2}\right)^2 \right\}$$

$$= \frac{f}{2I} \left(\frac{d^2}{4} - \frac{d^2}{4} \right) = 0 \text{ at end layer}$$

max^m shear stress at $y = 0$

$$\tau_{\max} = \frac{f}{2 \times \frac{bd^3}{12}} \times \left(\frac{d^2}{4} - 0 \right)$$
$$= \frac{3f}{bd^2} \times \frac{d^2}{4}$$

$$\tau_{\max} = \frac{3}{2} \frac{f}{bd} \quad \left(\text{max^m shear stress in rectangular section at section} \right)$$

→ Unit of $\tau_{\max}$ is N/mm^2

- (Q) A rectangular beam of width 10cm. is subjected to a max^m shear force of 50kN. And the max^m shear stress is 3N/mm^2 . Calculate the depth of beam. (previous yr qns) 5 marks.

Ans Given,

$$1\text{cm} = 10\text{mm}$$

$$b = 10\text{cm} = 10 \times 10 = 100\text{mm}$$

$$F = 50\text{kN} = 50 \times 10^3\text{N}$$

$$\tau_{\max} = 3\text{N/mm}^2$$

$$d = ?$$

$$\tau_{\max} = \frac{3F}{2bd}$$

$$\Rightarrow 3 = \frac{3 \times 50 \times 10^3}{2 \times 100 \times d}$$

$$\Rightarrow d = \frac{3 \times 50 \times 10^3}{2 \times 100 \times 3}$$

$$= \frac{10^3}{4} = 250\text{mm}.$$

(Q) A rectangular beam of width 100mm and depth 250mm is carrying an UDL of 40kN/m. Over a length of 3m. Calculate the max shear stress in the beam and draw the stress distribution diagram.

Any Given,

$$b = 100\text{mm}$$

$$d = 250\text{mm}$$

$$F = \frac{WL}{2} = \frac{40 \times 3}{2} = 60\text{kN}$$

$$= 60 \times 10^3 \text{ N}$$

$$W = 40\text{kN/m} = 40 \times 10^3$$

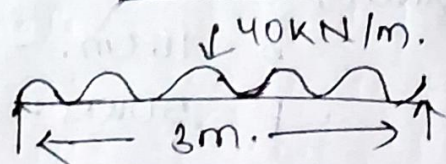
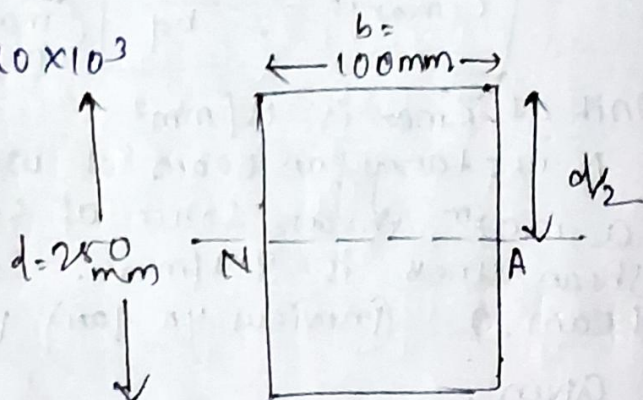
$$\tau_{\text{max}} = ?$$

$$\tau_{\text{max}} = \frac{3F}{2bd}$$

$$= \frac{3 \times 60 \times 10^3}{2 \times 100 \times 250}$$

$$= 3.6 \text{ N/mm}^2$$

$$= 3.6 \text{ N/mm}^2$$



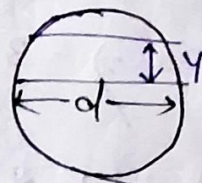
Simply supported beam under UDL at centre $= \frac{WL}{2}$

Circular Section →

d = diameter of circular section

r = radius = $d/2$

$$\tau = f \times \frac{r^2 - y^2}{3I}$$



$$\tau_{\text{max}} = f \times \frac{r^2 - 0}{3 \times \frac{\pi}{64} d^4}$$

$$\tau_{\text{max}} = \frac{f \times \left(\frac{d}{2}\right)^2}{3 \times \frac{\pi}{64} \times d^4}$$

$$\tau_{\text{max}} = f \times \frac{16}{3 \times \pi \times d^2 \times 4}$$

$$\tau_{\text{max}} = f \times \frac{16}{3 \pi d^2}$$

(Q) A circular beam of diameter 100mm is subjected to a shear force of 30kN. Calculate the max^m value of shear stress and draw the stress - distribution diagram?

Ans $d = 100\text{mm}$

$F = 30\text{ kN} = 30 \times 10^3\text{ N}$

$\tau_{\text{max}} = F \times \frac{16}{3\pi d^2}$

$= 30 \times 10^3 \times \frac{16}{3 \times \pi \times (100)^2}$

$= 5.09\text{ N/mm}^2$ Ans



(Q) A timber beam of 100mm width and 150mm depth supports an UDL over a span of 2m. If the safe stresses are 28 N/mm^2 in bending and 2 N/mm^2 in shear. Calculate the max^m load which the beam can carry? (previous yr qns)

Ans Given,

$\sigma = 28\text{ N/mm}^2$ (stress)

$\tau_{\text{max}} = 2\text{ N/mm}^2$ (shear)

$\tau_{\text{max}} = \frac{3F}{2bd}$

$2 = \frac{3F}{2bd}$

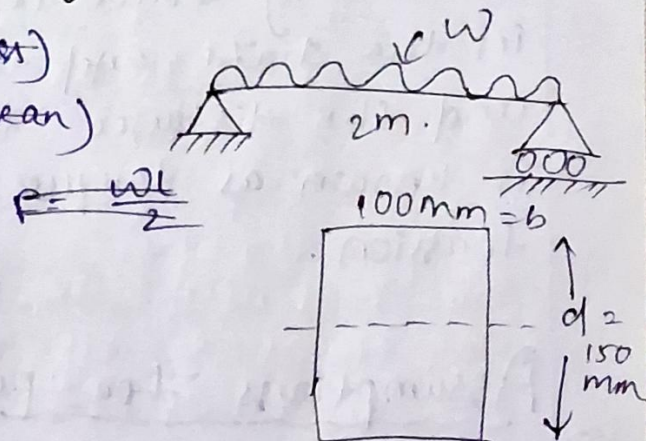
Given, $b = 100\text{mm}$

$d = 150\text{mm}$

$L = 2\text{m}$

$\sigma = 28\text{ N/mm}^2$

$\tau_{\text{max}} = 2\text{ N/mm}^2$



$F = \frac{wL}{2}$

max^m shear stress for rectangular section

$\tau_{\text{max}} = \frac{3F}{2bd}$

$2 = \frac{3}{2} \times \left(\frac{F}{100 \times 150} \right)$

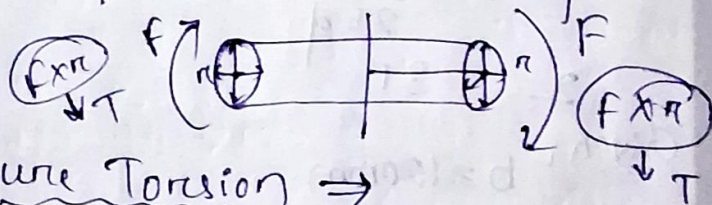
$2 = \frac{3F}{2(100 \times 150)}$

Torsion →

$$\frac{\sigma}{r} = \frac{\phi}{\rho}$$

21.04/12/22

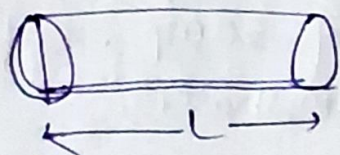
When a circular ~~shaft~~ shaft is subjected to a turning force it produces a twisting motion in the shaft and the product of ^{the} turning force and the distance from the centre of the shaft, is known as torque and the effect is called torsion.



Assumptions for pure Torsion ⇒

- ① The material of the shaft is uniform throughout.
- ② The twist along the shaft is uniform.
- ③ The plane section before bending remains plane after bending.
- ④ All the diameter of normal section, which were straight before twist remains straight after ~~the~~ twist.

Torsional stress in shaft →



θ = Angle of twist

L = length of shaft.

R = Radius of the shaft

T = Torque produced

G = modulus of rigidity.

$$\boxed{\frac{\tau}{R} = \frac{G \cdot \theta}{L}} \quad \text{IMP}$$

τ = Torsional stress

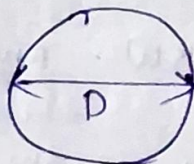
J = polar moment of Inertia

Torsion Equation →

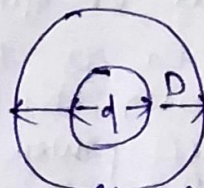
$$\boxed{\frac{\tau}{R} = \frac{G \cdot \theta}{L} = \frac{T}{J}} \quad \text{IMP}$$

Strength of solid shaft →

$$T = \frac{\pi}{16} \times \tau \times D^3$$



$$T = \frac{\pi}{16} \tau \times \left(\frac{D^4 - d^4}{D} \right)$$



Power transmitted by a shaft →

$$\boxed{P = \frac{2\pi NT}{60} \text{ KW}} \quad \text{IMP} \quad \text{L 5 marks q.}$$

N = revolution per minute

T = Torque in KN/m.

Q. 5 Marks Questions. A circular shaft of diameter 60mm 04.05/12/23
is running at a speed of ~~150~~ 150 rpm. If the shear stress does not exceed 50mp. Calculate the power transmitted by the shaft.

Ans diameter = 60mm (D) $\tau = 50\text{mp}$
 $N = 150\text{ RPM.}$

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$= \frac{\pi}{16} \times 50 \times (60)^3$$

$$= 2120575.04 \text{ N.mm.}$$

$$= 2.12 \text{ kN.m.}$$

$$N \rightarrow \frac{\text{KN.}}{10^3}$$

$$P = \frac{2\pi NT}{60}$$

$$= \frac{2\pi \times 150 \times 2.12}{60}$$

$$= 33.30 \text{ kW. } \underline{\text{Ans}}$$

(Q) Previous yr qns

A solid shaft of diameter 200mm. has the same crosssectional area as that of a hollow shaft of same material, with the inside diameter 150mm. Find the ratio of power transmitted by the two shaft at the same speed?

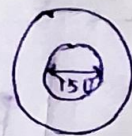
Ans

Given, $D_1 = 200\text{mm}$ (solid shaft)

$d_2 = 150\text{mm}$ (hollow shaft)

$D_2 = ?$

$$A_s = A_h$$



$$A_s = \frac{\pi}{4} D^4$$

$$A_h = \frac{\pi}{4} (D^4 - d^4)$$

$$\Rightarrow \frac{\pi}{4} D^4 = \frac{\pi}{4} (D^4 - d^4)$$

$$\Rightarrow D^4 = D^4 - d^4$$

$$\Rightarrow (200)^4 = D^4 - (150)^4$$

$$\Rightarrow D^4 = (200)^4 + (150)^4 = 2106250000$$

$$D^4 = 2106250000$$

$$D = \sqrt[4]{2106250000}$$

$$= 214.23 \text{ mm.}$$

Ans

Then,

$$\frac{P_s}{P_H} = \frac{\frac{2\pi NT_s}{60}}{\frac{2\pi NT_H}{60}} = \frac{T_s}{T_H}$$

$$\frac{P_s}{P_H} = \frac{T_s}{T_H} = \frac{\frac{\pi}{16} \times \tau \times D^3}{\frac{\pi}{16} \times \tau \times \left(\frac{D^4 - d^4}{D} \right)}$$

$$= \frac{(200)^3}{\frac{(214.23)^4 - (150)^4}{214.23}}$$

$$= 1.071 \text{ Ans}$$

(Q) Find the torque a solid shaft of 100mm diameter can transmit. If the max angle of twist is 1.5 degree in a length of 2m. and $G = 70 \text{ GPa}$.

Ans → Given,

$$D = 100 \text{ mm}$$

$$\theta = 1.5^\circ$$

$$L = 2 \text{ m} = 2000 \text{ mm}$$

$$G = 70 \text{ GPa} = 70 \times 10^3 \text{ MPa}$$

$$\frac{\tau}{R} = \frac{G\theta}{L} \quad T = ?$$

$$\Rightarrow \frac{\tau}{50} = \frac{70 \times 10^3 \times 1.5}{2000}$$

$$\Rightarrow \tau = \frac{70 \times 10^3 \times 1.5 \times 50}{2000} = 2625 \text{ N/mm}^2$$

$$T = \frac{\pi}{16} \times 2 \times D^3$$

$$= \frac{\pi}{16} \times 2625 \times (100)^3$$

$$= 515417544.72 \text{ N}\cdot\text{mm}$$

$$= 515.4 \text{ kN}\cdot\text{m}$$

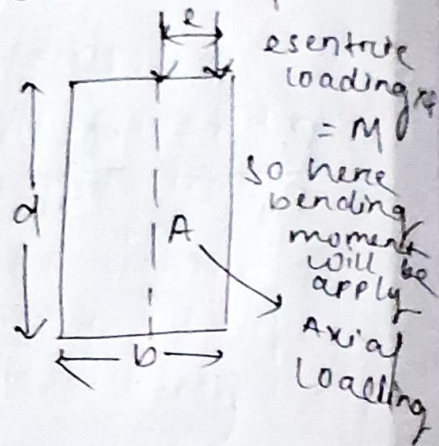
Ans

Combined Bending and direct stress → Q4.06/12/23

$$\sigma = \frac{P}{A}, \sigma_b = \frac{M}{I} y$$

$$\sigma_{\max} = \frac{P}{A} \left(1 + \frac{6e}{b} \right)$$

$$\sigma_{\min} = \frac{P}{A} \left(1 - \frac{6e}{b} \right)$$



Where, P = load

A = Area of cross-section

e = eccentricity

b = width of beam.

(Q) A rectangular column ~~is~~ of width 150 mm and thickness ~~150~~ 120 mm carries a load of 180 kN at an eccentricity of 10 mm. Calculate the intensity of max^m and min^m stress

Ans

Given,

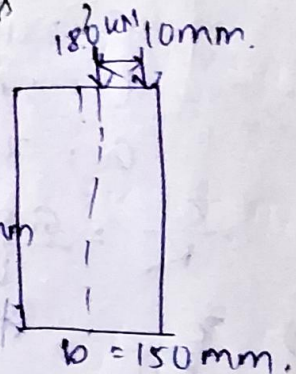
$$b = 150 \text{ mm}, A = 150 \times 120$$

$$= 18000 \text{ mm}^2$$

$$d = 120 \text{ mm}$$

$$P = 180 \text{ kN} = 180 \times 10^3 \text{ N} = 120 \text{ mm}$$

$$e = 10 \text{ mm}$$



$$\sigma_{\max} = \frac{P}{A} \left(1 + \frac{6e}{b} \right)$$

$$= \frac{180 \times 10^3}{18000} \left(1 + \frac{6 \times 10}{150} \right)$$

$$= 10(1 + 0.4)$$

$$= 14 \text{ N/mm}^2$$

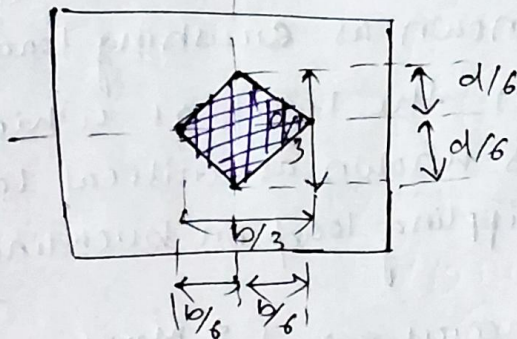
$$\sigma_{\min} = \frac{P}{A} \left(1 - \frac{6e}{b} \right)$$

$$= \frac{180 \times 10^3}{18000} \left(1 - \frac{6 \times 10}{150} \right)$$

$$= 10(1 - 0.4)$$

$$= 6 \text{ N/mm}^2$$

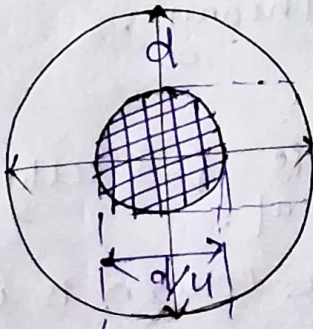
Limit of eccentricity $\rightarrow$
for rectangular section $\rightarrow$



$$e \leq \frac{b}{6} \text{ or } \frac{d}{6}$$

Cone / kernel of the section.

for circular section $\rightarrow$



$$e \leq \frac{d}{8}$$

Cone / kernel of the section.

Column and Strut →

DL 07/12/23

The ^{members} ~~mainwork~~ in a civil Engg. structure which ~~test the~~ ^{test} the compressive load which is known as ~~strut~~ ^{column}.
(vertical) →
(column, wall)



- A strut can be horizontal, inclined and vertical.
- The vertical strut is known as column.

Crushing load and critical load / crippling load →

- For a short column the load at which the column fails is known as crushing load.
- For a long column the load at which the column buckles is known as critical load or ~~buckling load~~ crippling load or buckling load.

Euler's Column Theory → 5 Marks.

To determine the value of critical load for long columns Euler has proposed a theory which is known as Euler's Column Theory.

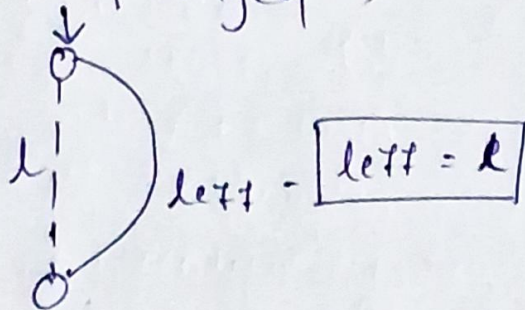
Assumptions →

- Initially the column is perfectly straight and the load applied is axial.
- The cross-section of the column is uniform, throughout its length.
- The column material is perfectly elastic, homogeneous and isotropic and obeys Hooke's law.
- The length of the column is very large as compared to its cross-sectional dimension.
- The ~~short~~ shortening ~~the~~ of column due to compression is neglected.

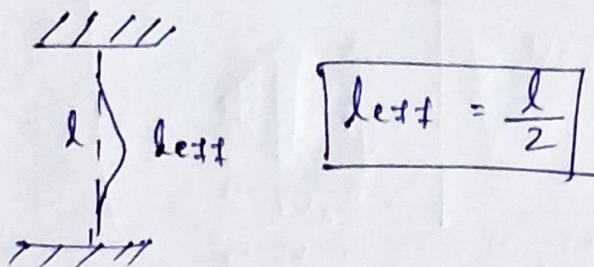
→ The failure of column occurs due to buckling only.

End conditions of column →

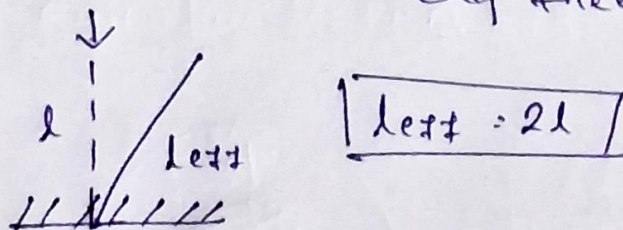
(a) Both end hinged →



(b) Both end fixed →



(c) One end fixed and other ~~fixed~~ end free →



(d) One end fixed and other end hinged →

